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Verification of two- and three-parameter simple height-diameter models for birch in the European part of Russia

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Abstract: The accuracy of determining the height of trees is essential both in forestry and in scientific research. Height is usually determined using specific models, where it is a function of the diameter at breast height. On the materials of 23 sample plots with the measurement of model trees in birch stands, the parameters were determined for 29 two--parameter and three-parameter models that are most often found in literary sources. The following metrics evaluated the quality of the models: root mean square error, mean absolute percentage error, coefficient of determination, adjusted coefficient of determination, Akaike information criterion, and Bayesian information criterion. Three-parameter models of the dependence of height on diameter by a set of metrics show somewhat better quality than two-parameter models. Nevertheless, in general, the differences between most models are minor. Along with the models selected as the best, the Näslund and Chapman-Richards equations, which are often used in the literature as the most flexible, showed good quality. The methodology of this study allows you to repeat the same work for tree species and forest conditions, for which information on the nature of the relationship of height with diameter is incomplete or missing.

Keywords: simple model; selection of models

Diameter at breast height (DBH) and tree height (h) are important characteristics used in forestry. Diameter at breast height and height are considered the main variables for determining trunk volumes and biomass. The physicochemical properties of wood, for example its lignin and cellulose content, density, are determined by the diameter and height of the trunks (Kroon 2008). From the graphical dependences of the height on the diameter of the trees, the average height corresponding to the average tree in the stand is determined.

Compared to the diameter, the height of the tree is much more challenging to determine, the measurement process takes longer, and the obtained values can deviate significantly from the actual ones (Colbert et al. 2002). Therefore, heights are not usually measured for all trees on the site. When carrying out an inventory of forests according to the results of selective measurements, heights are usually calculated according to the pairwise dependence on the diameters of the trees. Simultaneously, the question of the accuracy of models that convey the relationship between the heights and diameters of trees is of particular relevance. This issue is debatable in the literature (Lei et al. 2009; El Mamoun et al. 2013; Mehtätalo et al. 2015).

Simple models express height as a function of diameter only at breast height. Many simple models of complexity are classified into two-parameter, three-parameter, multiparameter ones. Three-parameter

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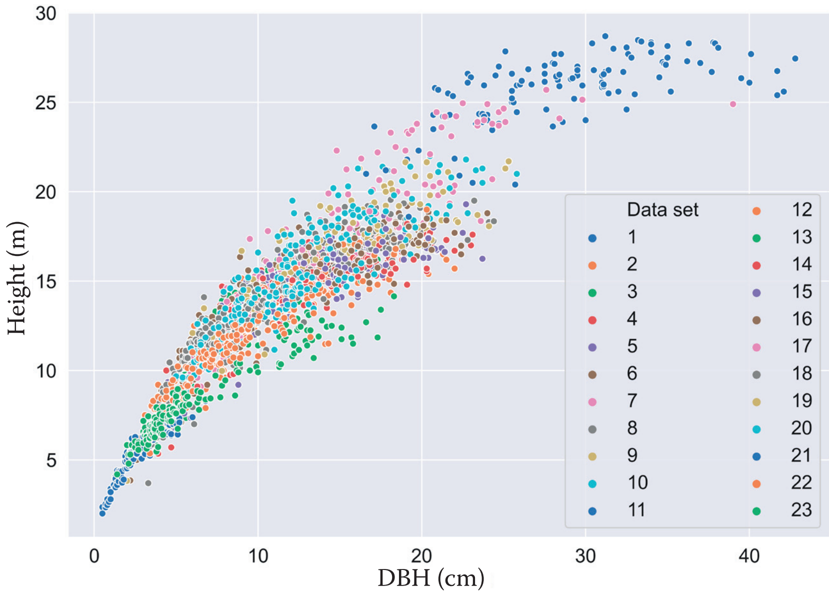

Figure 1. Tree height and diameter at breast height (DBH) data for model fitting

models, in comparison with two-parameter ones, are more flexible and allow for a more detailed transmission of the dependence. However, this is not always the case. For example, Mehtätalo et al. (2015), according to the results of fitting 16 nonlinear functions from 28 data sets for different tree species and regions, showed that two-parameter models were preferred in most cases. The problem of using three-parameter models is that the solution does not converge when fitting the coefficients (Mehtätalo et al. 2015; Ogana, 2018). Be that as it may, this problem is solved by setting a fixed value of one of the parameters, which affects the accuracy of the obtained dependence.

Simple functions that reflect the pairwise relationship between the heights and diameters at the local level are the basis for the development of generalised models. At the stage of specification of generalised models, it is essential to include simple dependences in their basis, which can be considered the best according to a set of criteria. The aim of the present study is to select the most appropriate one from the set of simple models from the materials used to measure model trees in birch stands that convey the relationship between the height of the trees and the diameter at breast height.

MATERIAL AND METHODS

Study site. The Forest Experimental District of the Russian State Agrarian University – Moscow Timiryazev Agricultural Academy is located in the central part of the East European Plain. The area of the Forest Experimental District is 248.6 hectares. The climate is temperate continental with an average annual temperature of 6.1 °C (for 1987–2016) and average annual precipitation of 700 mm (for 1987–2016). Soils are mainly sod-podzolic. The predominant tree species are pine, larch, oak, birch, linden, and maple (Dubenok et al. 2020).

In 1887, the area with birch forests was 42 ha, then it decreased as a result of logging to 26 ha in 1935–1945 and then it expanded after the decay of natural pine forests to 50 ha in 1987–2009. The stock increased with some delay and amounted to about 7 thousand m^3 in 1915–1935. In 1955, it decreased to 3.9 thousand m^3, then, following an increase in the area, the stock began to grow, amounting to 13.3 thousand m^3 in 1987. After the 1998 hurricane, the stock was reduced to 10.5 thousand m^3 and the average wood stock in 2009 was 210 $m^3 \cdot ha^{-1}$.

Data collection. The study uses data collected on 23 sample plots (from 0.2 to 0.5 ha). The age of the stands in which the model trees were measured was from 10 to 85 years. The average diameter was from 3 to 30 cm, and the average height was from 6 to 27 m. In the experimental plots, 35 to 153 trees were measured. A total of 2201 trees were measured with a diameter of 0.5 to 42.1 cm and a height of 2.0 to 28.7 m (Figure 1). Tree heights of young stands were measured with a measuring tape from the root collar using a ladder. In middle-aged and ripening plants, every second row was cut down. In

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mature forest stands, all the trees were cut down. In felled trees, biometric indicators were measured.

Height-diameter models. Generally, the relationship between the height and diameter of a tree is nonlinear and height curve increases more rapidly in earlier stages than in later stages (Lappi 1997; Pretzsch 2009; Schmidt et al. 2011; Sharma et al. 2016). Our data also shows a significant nonlinear pattern (Figure 1). Twenty-nine candidate models were selected from previous studies based on their appropriate mathematical features, possible biological interpretation of parameters, and satisfactory prediction for a tree height-diameter relationship in the literature (Table 1). Selected models are classified as follows: (*i*) two-parameter models (M1-M15); (*ii*) three-parameter models (M16-M29).

The coefficients of 29 height-diameter models were adjusted for each of the 23 sample plots.

Data analysis. The nonlinear least-squares method was used to fit functions. The trust region reflective algorithm and the dogleg algorithm with rectangular trust regions were used to optimise the objective function. To select models that better describe the relationship between the heights and diameters of the trees, six metrics were used:

(*i*) Root mean square error (*RMSE*);
(*ii*) Mean absolute percentage error (*MAPE*);
(*iii*) Coefficient of determination (R^2);
(*iv*) Adjusted coefficient of determination (R^2-adj.);
(*v*) Akaike information criterion (*AIC*);
(*vi*) Bayesian information criterion (*BIC*).

The equations of these metrics are summarised in Table 2. For all samples, the average value of the metrics was calculated. The macro average is calculated as the arithmetic average. The weighted average value is calculated to eliminate imbalance between samples, considering the number of observations in each sample. In general, models with the lowest averages of *RMSE*, *MAPE*, *AIC* and BIC and with the highest averages of R^2 and R^2-adj. are recognised as the best (Aertsen et al. 2010; Ahmadi et al. 2013; Chai et al. 2018). All analyses of data were performed using Python version 3.5 and Pandas, NumPy, SciPy, scikit-learn packages. The figures were drawn using the seaborn package.

RESULTS AND DISCUSSION

The functions describing the relationship between height and diameter are subject to certain requirements. Firstly, the intercept of the equation should have a value of 1.3. Secondly, the curve should be increasing and have a horizontal asymptote. For all the models considered in the study, the first requirement is satisfied. Given the selected coefficients, the models M10, M17, M23, M26 do not satisfy the second requirement, and are therefore excluded from further consideration.

The final averaged quality estimates of the models are shown in Table 3. For a set of metrics, the best quality of the two-parameter models was shown by the M11 and M12 models (macro-average *RMSE* = 0.915, *MAPE* = 5.378, R^2 = 0.819, R^2-adj. = 0.813, *AIC* = –19.4 and BIC = –14.5). The worst quality of the two-parameter models was found in the M1 and M15 models (macro-average *RMSE* = 1.043, *MAPE* = 6.343, R^2 = 0.774, R^2-adj. = 0.768, *AIC* = 8.0 and BIC = 12.9). The frequently used Näslund equation (M2) showed a good result (macro-average *RMSE* = 0.927, *MAPE* = 5.451, R^2 = 0.815, R^2-adj. = 0.809, *AIC* = –16.4 and BIC = –11.5). In general, all two-parameter models, except M1, M6, M13, M15 models, showed approximately the same values of quality metrics.

For a set of metrics, the best quality of the three-parameter models was shown by the M17 (macro-average *RMSE* = 0.894, *MAPE* = 5.241, R^2 = 0.827, R^2-adj. = 0.819, *AIC* = –21.2 and BIC = –13.8) and M28 model (macro-average *RMSE* = 0.894, *MAPE* = 5.238, R^2 = 0.827, R^2-adj. = 0.820, *AIC* = –21.2 and BIC = –13.8). The worst quality of the three-parameter models was shown by the M22 model (macro-average *RMSE* = 1.043, *MAPE* = 6.343, R^2 = 0.774, R^2-adj. = 0.764, *AIC* = 10.0 and BIC = 17.4). The frequently used Chapman-Richards function (M20), with a clear interpretation of the parameters, showed a good result (macro-average *RMSE* = 0.895, *MAPE* = 5.257, R^2 = 0.827, R^2-adj. = 0.819, *AIC* = –21.1 and BIC = –13.7). In general, all three-parameter models, except M22 model, showed approximately the same values of quality metrics.

With further examination of the predictive ability of the models, the 45-degree line plots were produced for M12 model (Figure 2A) and M28 model (Figure 2B). As can be seen, these models tended to make an angle of 45 degrees with the axis, meaning there was no significant difference between the measured and the predicted values. For the most part, three-parameter models showed better quality than two-parameter models, but the differences between them are minor. Choosing a specific model from all their diversity is a

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Table 1. The applied *H*-DBH functions

ID	Equation	References
M1	$h=1.3+b_1 DBH^{b_2}$	Huxley and Teissier (1936)
M2	$h=1.3+\left(\frac{DBH}{b_1+b_2 DBH}\right)^2$	Näslund (1936); Mehtätalo et al. (2015)
M3	$h=1.3+\frac{b_1 DBH}{b_2+DBH}$	Huang et al. (2000)
M4	$h=1.3+b_1\left(\frac{DBH}{1+DBH}\right)^{b_2}$	Huang et al. (2000)
M5	$h=1.3+b_1\left(1+\frac{1}{DBH}\right)^{b_2}$	Curtis (1967)
M6	$h=1.3+\frac{b_1 DBH}{(1+DBH)^{b_2}}$	Curtis (1967)
M7	$h=1.3+b_1(1-\exp(-b_2 DBH))$	Meyer (1940)
M8	$h=1.3+\exp\left(b_1+\frac{b_2}{DBH+1}\right)$	Wykoff et al. (1982)
M9	$h=1.3+\frac{b_1 DBH}{(DBH+1)+b_2 DBH}$	Bates and Watts (1980)
M10	$h=1.3+b_1 DBH\exp(-b_2 DBH)$	Huang et al. (2000)
M11	$h=1.3+\exp\left(b_1+\frac{b_2}{DBH}\right)$	Staudhammer and LeMay (2000)
M12	$h=1.3+b_1\exp\left(\frac{b_2}{DBH}\right)$	Buford (1986)
M13	$h=1.3+b_1(\ln(1+DBH))^{b_2}$	El Momoun et al. (2013)
M14	$h=1.3+\left(b_1+\frac{b_2}{DBH}\right)^{-5}$	El Momoun et al. (2013)
M15	$h=1.3+\left(\left(\frac{b_1}{DBH}\right)^{b_2}\right)^{-1}$	Ogana (2018)
M16	$h=1.3+\frac{b_1}{1+b_2 DBH^{-b_3}}$	Huang et al. (2000)
M17	$h=1.3+\frac{DBH^2}{b_1+b_2 DBH+b_3 DBH^2}$	Huang et al. (1992)
M18	$h=1.3+\frac{b_1}{1+b_2\exp(-b_3 DBH)}$	Huang et al. (1992)
M19	$h=1.3+b_1(1-\exp(-b_2 DBH^{b_3}))$	Yang et al. (1978)
M20	$h=1.3+b_1(1-\exp(-b_2 DBH))^{b_3}$	Peng et al. (2001)
M21	$h=1.3+b_1\exp(-b_2\exp(-b_3 DBH))$	Huang et al. (1992)
M22	$h=1.3+\exp(b_1+b_2 DBH^{b_3})$	Larsen and Hann (1987)
M23	$h=1.3+\exp\left(b_1+\frac{b_2}{DBH+b_3}\right)$	Ratkowsky (1990)
M24	$h=1.3+b_1\exp(-b_2 DBH^{-b_3})$	Stage (1963)
M25	$h=\left(1.3^{b_1}+\left(b_2^{b_1}-1.3^{b_1}\right)\frac{1-\exp(-b_3 DBH)}{1-\exp(-100 b_3)}\right)^{\frac{1}{b_1}}$	Schnute (1981)
M26	$h=1.3+b_1\sqrt{DBH}+b_2 DBH+b_3 DBH^2$	Atroshchenko (2004)
M27	$h=1.3+\frac{b_1}{1+\left(b_2 DBH^{b_3}\right)^{-1}}$	Peschel (1938)
M28	$h=1.3+b_1 DBH^{b_2 DBH^{-b_3}}$	Sibbesen (1981)
M29	$h=1.3+DBH^{\frac{b_1}{b_2+b_3 DBH^{b_1}}}$	El Momoun et al. (2013)

h – tree height (m); DBH - diameter at breast height (cm); b_1, b_2, b_3 – model parameters

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Table 2. Model performance criteria selected for this study

ID	Function name	Equation
1	Root mean square error (*RMSE*)	$RMSE = \sqrt{\sum \frac{(y_i - \hat{y}_i)^2}{n}}$
2	Mean absolute percentage error (*MAPE*)	$MAPE = 100 \times \sum \left\lvert \frac{y_i - \hat{y}_i}{y_i} \right\rvert / n$
3	Coefficient of determination (R^2)	$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \acute{y})^2}$
4	Adjusted coefficient of determination (R^2-*adj.*)	$R^2_{adj.} = 1 - (1 - R^2)\frac{(n-1)}{(n-k)}$
5	Akaike information criterion (*AIC*)	$AIC = 2k + n \ln \frac{\sum (y_i - \hat{y}_i)^2}{n}$
6	Bayesian information criterion (*BIC*)	$BIC = k \ln n + n \ln \frac{\sum (y_i - \hat{y}_i)^2}{n}$

k – number of model parameters; n – number of observations; y_i – measured value; $\hat{y}_i$ – predicted value

Table 3. Model quality assessment

ID	Macro-average						Weighted average					
	RMSE	*MAPE*	R^2	R^2-*adj.*	*AIC*	*BIC*	*RMSE*	*MAPE*	R^2	R^2-*adj.*	*AIC*	*BIC*
M1	1.043	6.343	0.774	0.768	8.0	12.9	1.078	6.735	0.788	0.783	17.5	22.8
M2	0.927	5.451	0.815	0.809	−16.4	−11.5	0.948	5.711	0.830	0.826	−13.1	−7.8
M3	0.954	5.699	0.806	0.801	−10.1	−5.2	0.980	6.016	0.821	0.816	−4.9	0.4
M4	0.917	5.371	0.818	0.813	−18.8	−13.9	0.936	5.604	0.834	0.830	−16.4	−11.1
M5	0.917	5.371	0.818	0.813	−18.8	−13.9	0.936	5.604	0.834	0.830	−16.4	−11.1
M6	1.021	6.171	0.782	0.776	3.3	8.2	1.054	6.547	0.796	0.792	11.7	17.0
M7	0.925	5.517	0.817	0.811	−16.1	−11.2	0.949	5.806	0.831	0.827	−12.4	−7.1
M8	0.923	5.436	0.816	0.811	−16.9	−12.0	0.944	5.688	0.832	0.828	−13.9	−8.6
M9	0.954	5.699	0.806	0.801	−10.1	−5.2	0.980	6.016	0.821	0.816	−4.9	0.4
M10	0.911	5.431	0.821	0.816	−19.1	−14.1	0.934	5.701	0.835	0.832	−16.3	−11.0
M11	0.915	5.378	0.819	0.813	−19.4	−14.5	0.933	5.597	0.835	0.831	−17.4	−12.1
M12	0.915	5.378	0.819	0.813	−19.4	−14.5	0.933	5.597	0.835	0.831	−17.4	−12.1
M13	0.988	5.905	0.794	0.788	−3.4	1.5	1.016	6.243	0.809	0.804	3.3	8.6
M14	0.917	5.422	0.819	0.813	−19.1	−14.2	0.933	5.638	0.835	0.831	−17.3	−12.0
M15	1.043	6.343	0.774	0.768	8.0	12.9	1.078	6.735	0.788	0.783	17.5	22.8
M16	0.895	5.246	0.827	0.819	−21.1	−13.7	0.915	5.482	0.841	0.836	−19.5	−11.5
M17	0.894	5.241	0.827	0.819	−21.2	−13.8	0.914	5.477	0.842	0.836	−19.6	−11.7
M18	0.903	5.365	0.824	0.816	−18.7	−11.3	0.925	5.624	0.838	0.832	−16.6	−8.7
M19	0.894	5.264	0.827	0.819	−21.0	−13.6	0.915	5.507	0.841	0.836	−19.4	−11.4
M20	0.895	5.257	0.827	0.819	−21.1	−13.7	0.915	5.496	0.841	0.836	−19.5	−11.5
M21	0.897	5.293	0.826	0.818	−20.4	−13.1	0.917	5.539	0.840	0.835	−18.7	−10.8
M22	1.043	6.343	0.774	0.764	10.0	17.4	1.078	6.735	0.787	0.780	19.5	27.4
M23	0.900	5.260	0.826	0.818	−20.0	−12.6	0.921	5.500	0.840	0.834	−18.0	−10.0
M24	0.899	5.258	0.826	0.818	−20.3	−12.9	0.919	5.496	0.840	0.835	−18.4	−10.5
M25	0.894	5.265	0.827	0.819	−21.1	−13.7	0.915	5.507	0.841	0.836	−19.5	−11.5
M26	0.910	5.412	0.822	0.814	−16.5	−9.1	0.935	5.694	0.835	0.829	−13.2	−5.3
M27	0.895	5.246	0.827	0.819	−21.1	−13.7	0.915	5.482	0.841	0.836	−19.5	−11.5
M28	0.894	5.238	0.827	0.820	−21.2	−13.8	0.914	5.476	0.842	0.836	−19.6	−11.6
M29	0.898	5.391	0.825	0.817	−18.7	−11.3	0.919	5.638	0.839	0.833	−16.9	−8.9

RMSE – Root mean square error; *MAPE* – Mean absolute percentage error; R^2 – Coefficient of determination; R^2-adj. – Adjusted coefficient of determination, *AIC* – Akaike information criterion; *BIC* – Bayesian information criterion

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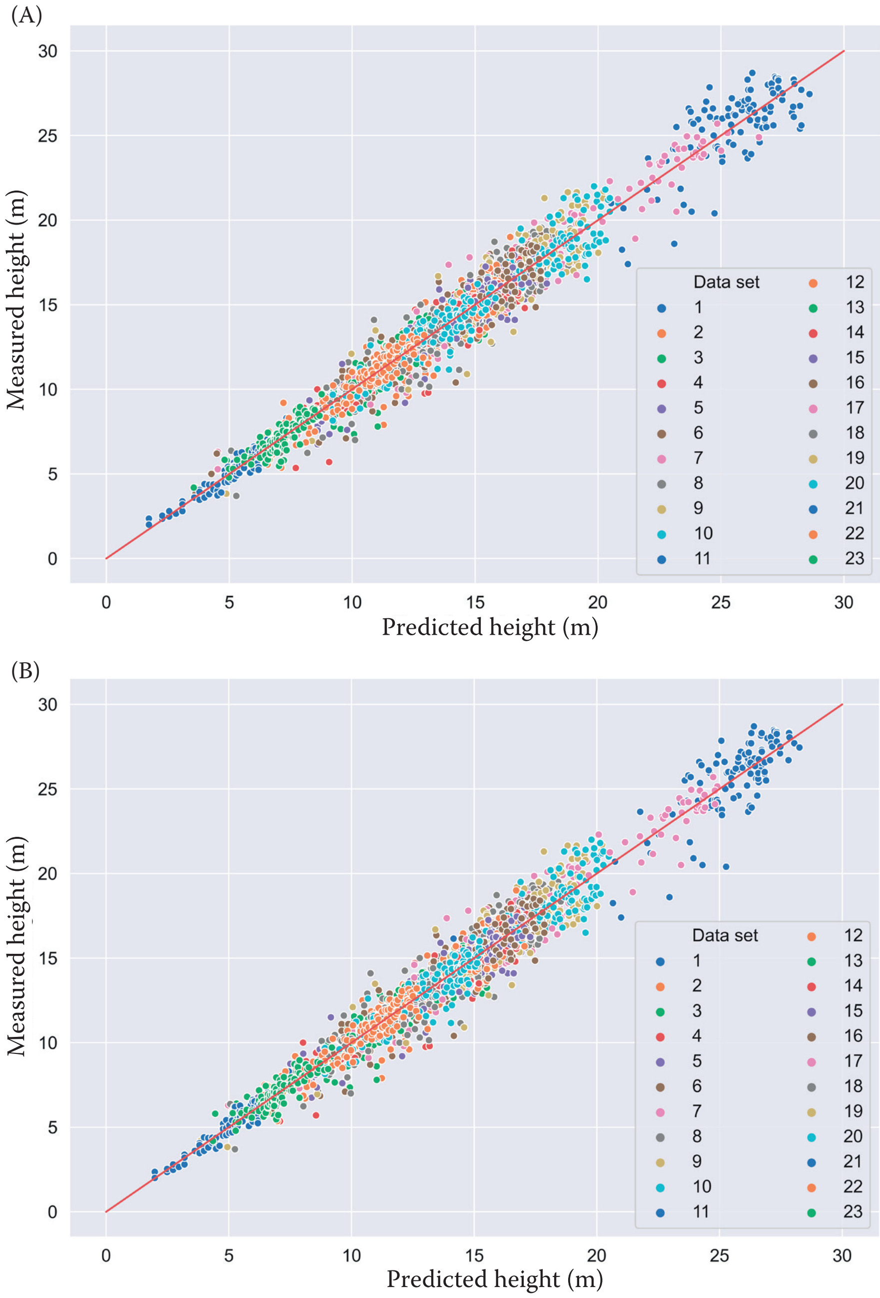

Figure 2. Plot of observed values versus predicted values for model M12 (A), M28 (B), red line represents the diagonal

difficult task, because the quality metrics for most of them are close to each other. Thus, the selection of a specific best model can be considered subjective to some extent. In many studies, the equations of Näslund (M2) or Chapman-Richards (M20) are chosen as models of the dependence of height on the diameter at breast height, and they are characterised as the most flexible [Kangas, Maltamo 2002; Sharma, Parton 2007; Jiang, Li 2010; Mehtätalo et al. 2015]. The results of this study confirm the appropriateness of using these equations.

The results cannot be extended to other tree species and regions, because the biological characteristics of species and growing conditions can have a significant impact on the shape of the relationship curve between height and diameter. On the example of stands of red acacia (*Vachellia seyal*), sterculia (*Sterculia setigera*), Egyptian balsam (*Balanites aegyptiaca*), African birch (*Anogeissus leocarpus*), combretum (*Combretum hartmannianum*), terminalia (*Terminalia brownii*) in the Blue Nile State Reserve, and Sudan (El Mamoun et al. 2013) it was

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shown that the choice of a particular model depends on the tree species. Studies carried out for stands of Turkish pine (*Pinus brutia*), black pine (*Pinus nigra*), and Lebanese cedar (*Cedrus libani*) in southern Turkey (Özçelik et al. 2014) showed that the height-diameter models were significantly different for disparate environmental regions. The authors concluded that, to increase the accuracy of forecasting, such models should be developed for individual ecoregions.

CONCLUSION

Justified as the best models of the dependence of height on diameter at breast height can be used in practice when performing forestry and research work in birch forest stands in the central regions of the European part of Russia. Three-parameter models of the dependence of height on diameter by a set of metrics (*RMSE*, *MAPE*, R^2, R^2-adj., *AIC*, *BIC*) show somewhat better quality than two-parameter models. Notwithstanding, predominantly, the differences between most models are insignificant. The methodology of this study allows you to repeat the same work for tree species and forest conditions, for which information on the nature of the relationship of height with diameter is incomplete or missing.

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