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Evaluation of four methods of fitting Johnson's S_{BB} for height and volume predictions

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Abstract

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Johnson's S_{BB} is the most commonly used bivariate distribution model in forestry. There are different methods of fitting Johnson's distribution, and their accuracies differ. In this article, the method of conditional maximum likelihood (CML), moments, mode and Knoebel and Burkart (KB) were used to fit Johnson's S_{BB} distribution. A total of 4,237 diameter and height data obtained from 90 sample plots of *Eucalyptus camaldulensis* Dehnhardt were used. Evaluation was based on tree height and volume predictions. The predicted and observed tree heights and volumes were compared using the paired sample *t*-test. The average relative (%) bias and root mean square error of heights and volumes were computed for the four methods. The results showed that CML- and moments-based methods were more suitable than KB and mode methods for predicting tree height and volume. The level of significance and percentage bias were much lower in CML and moments. The mode-based method had the worst performance. The ranking order was: CML ≈ moments > KB > mode.

Keywords: conditional maximum likelihood; moments; mode; Knoebel and Burkhart; *Eucalyptus camaldulensis*

Tree diameter and height are important variables that determine the structure of forest stands. They are the fundamental tree characteristics from which other stand variables such as volume, density, basal area, site index (or average dominant height), etc., are derived. Tree diameter and height structures are usually expressed as frequency distribution of diameter or height that quantifies their distributions in diameter or height classes (Petráš et al. 2010). The representation of the joint distribution of tree diameter and height is termed bivariate distribution modelling. This provides a detailed and complex view of the forest (Rupšys, Petrauskas 2010). More so, it provides another means of improving the stand volume estimation (Mønness 2015).

Height-diameter models are routinely developed in tree distribution studies so that mean tree height can be predicted. This information is used to compute volume per diameter class by substituting the predicted mean tree height and the diameter class midpoint in a tree volume equation. However, the idea of using predicted height in the volume equation rather than actual height tends to neglect the fact that tree height can vary noticeably for a given diameter. As such, it initiates biases into the stand volume estimation (Schreuder, Hafley 1977). Studies have shown that the construction of height-diameter models through conditional distribution of height (i.e. bivariate distribution modelling) can help to reduce this bias effect and consequently, improve the stand volume estimation (Schreuder, Hafley 1977; Omule 1984; Tewari, von Gadow 1999; Li et al. 2002; Mønness 2015).

Several researchers have sometimes used different bivariate distributions in quantitative forestry for the improved stand volume estimation. Some of these distributions include: Johnson's S_{BB}

(Schreuder, Hafley 1977), bivariate lognormal (Nanang 2002), bivariate generalized beta (Li et al. 2002), bivariate logit-logistic (Wang, Rennolls 2007), bivariate power-normal (Mønness 2015), etc. To date, the bivariate Johnson's S_{BB} is the most commonly used bivariate distribution in forestry. It has flexible marginal distribution that is capable of describing univariate diameter and height distributions, able to accommodate both positive and negative skewness, provides a reasonable biological relationship between height and diameter in a simple median regression model and simplicity of parameter estimation.

There are different estimation methods of the univariate Johnson's S_B distribution: conditional maximum likelihood (CML), moments, mode, linear regression, Knoebel and Burkhart (KB) methods, etc. A number of studies have used some of these methods to fit the bivariate Johnson's S_{BB} distribution. For example, Schreuder and Hafley (1977), Omule (1984), Tewari and von Gadow (1999) utilised the CML method to fit the joint distribution of diameter and height. Li et al. (2002) used the KB method to fit bivariate S_{BB} distribution to the joint distribution of diameter and height data of Douglas-fir stands. The relative performance of these estimation methods differs considerably. The result obtained by Gorgoso-Varela and Rojo-Alboreca (2014) on the comparison of CML, moments, mode and KB revealed that the method of moments and CML performed better than the other methods for birch and pedunculate oak. Nevertheless, the method of moments has never been used to fit the bivariate Johnson's S_{BB} distribution of diameter and height to the best of my knowledge. The linear regression method is not frequently used because it requires considerable time during the model-fitting process (Zhou, McTague 1996). Therefore, the main objective of this study was to evaluate different estimation methods of the Johnson's S_{BB} distribution for tree height and volume predictions.

METHODS

Data

Data for this study came from the *Eucalyptus camaldulensis* Dehnhardt stands in Afaka Forest Reserve, Nigeria. The reserve lies between latitude 10.58°N–10.60°N and longitude 7.35°E–7.37°E with an elevation of 610 m a.s.l. The plantation covers an area of about 2,700 ha. Data were collected from 90 sample plots, each of 0.0625 ha in size. A total of 4,237 diameter and height measurements were available for analysis. The following stand variables were calculated from the inventory data: age, density, quadratic mean diameter, mean height, dominant height, growing space (i.e. the square root of the ratio of the square meter area of a hectare to the surviving number of stems per hectare), relative spacing, basal area and volume, etc. The statistics are presented in Table 1.

Johnson's univariate and bivariate distribution

The 4-parameter Johnson's S_B probability density function (Johnson 1949a) is expressed as Eq. 1:

$$f(x) = \frac{\delta}{\sqrt{2\pi}} \times \frac{\lambda}{(\xi+\lambda-x)(x-\xi)} \times e^{-\frac{1}{2}\left[\gamma+\delta\times\ln\left(\frac{x-\xi}{\xi+\lambda-x}\right)\right]^2} \quad (1)$$

where:

δ – kurtosis parameter, δ > 0,

λ – scale parameter, λ > 0,

Table 1. Statistics of stand variables (No. of plots = 90)

	Statistics			
	mean	maximum	minimum	SD
DBH (cm)	10.3	47.2	2.0	6.2
Height (m)	12.3	39.6	2.1	6.1
Age (yr)	23.1	29	7	8.4
Quadratic mean	11.8	23.9	5.9	3.8
Dominant height (m)	21.0	30.6	9.0	5.4
Density (No. per hectare)	753.0	1328	448.0	202.8
Growing space	3.74	4.72	2.74	0.49
Relative spacing	0.19	0.45	0.10	0.07
Basal area ($m^2 \cdot ha^{-1}$)	8.52	27.38	1.73	4.92
Volume ($m^3 \cdot ha^{-1}$)	144.54	520.92	12.22	94.81

SD – standard deviation

ξ – location parameter, $-\infty < \xi < +\infty$,
x – tree diameters and heights, $\xi < x < \xi + \lambda$,
γ – asymmetry parameter, $-\infty < \gamma < +\infty$.

The Johnson's S_{BB} (Johnson 1949b) is simply the bivariate extension of the univariate Johnson's S_B distribution, expressed as Eq. 2:

$$f\left(Z_d, Z_h, \rho\right) = \frac{1}{\left(2\pi\sqrt{1-\rho^2}\right)} \exp$$

$$\exp\left[-\frac{1}{2}\left(1-\rho^2\right)^{-1}\left(Z_d^2 - 2\rho Z_d Z_h + Z_h^2\right)\right] \quad (2)$$

where:

$$Z_d = \gamma_d + \delta_d \ln\left(\frac{d-\xi_d}{\xi_d + \lambda_d - d}\right),$$

$$Z_h = \gamma_h + \delta_h \ln\left(\frac{h-\xi_h}{\xi_h + \lambda_h - h}\right),$$

d – distribution of diameter,
h – distribution of height,
ρ – dependence measure between Z_d and Z_h,

$$\hat{\rho} = \sum_{i=1}^{n} Z_{di} Z_{hi} / n,$$

n – number of observation.

Johnson's S_{BB} height-diameter (*H-D*) median regression model

One major property of the S_{BB} is the regression relationship between diameter and height. The mean regression of S_{BB} is not usually used because it is complicated, however, the median regression takes a much simpler form; it is expressed as Eq. 3:

$$H = \frac{\hat{\lambda}_h \theta}{\left[\left(\frac{\hat{\xi}_d + \hat{\lambda}_d - D}{D - \hat{\xi}_d}\right)^{\phi} + \theta\right]} + \hat{\xi}_h \quad (3)$$

where:
H – total tree height (m),

$$\theta = \exp\left[\frac{\hat{\rho}\hat{\gamma}_d - \hat{\gamma}_h}{\hat{\delta}_h}\right],$$

$\phi = \frac{\hat{\rho}\hat{\delta}_d}{\hat{\delta}_h}$, $\phi > 0$ if $\rho > 0$ which may be assumed for diameters and heights,
$\hat{\lambda}_h, \hat{\xi}_h, \hat{\gamma}_h, \hat{\delta}_h, \hat{\lambda}_d, \hat{\xi}_d, \hat{\gamma}_d, \hat{\delta}_d$ – estimated parameters of the marginal distributions of heights and diameters, respectively,
D – diameter at breast height (cm).

The shape of the regression curve is influenced by ϕ and the slope depends on the value of ρ. The relationship is said to be linear if $\hat{\rho}\hat{\delta}_d = \hat{\delta}_h$ and $\hat{\rho}\hat{\gamma}_d = \hat{\gamma}_h$. This *H-D* model has been applied to estimate height for a given diameter. It has been reported that tree height can vary considerably for a given diameter and the S_{BB} *H-D* model is able to capture these variations compared to most traditional *H-D* models (Tewari, von Gadow 1999).

Fitting methods

Four fitting methods were used to estimate the parameters of Johnson's S_{BB} distribution wherein the *H-D* median regression model was fitted. These methods include: moments, CML, KB and the mode method. Each method was fitted to the marginal diameter and height.

Method of moments. This is based on the relationship between the parameters of Johnson's S_B distribution and the first and second moments of the marginal distributions of diameter and height (i.e. the mean and variance, respectively). This method was used by Fonseca et al. (2009), Gorgoso-Varela and Rojo-Alboreca (2014) and Ogana et al. (2017); it is expressed as Eqs 4–7:

$$\delta = \frac{\mu(1-\mu)}{SD(x)} + \frac{SD(x)}{4}\left[\frac{1}{\mu(1-\mu)} - 8\right] \quad (4)$$

$$\gamma = \delta \ln\left(\frac{1-\mu}{\mu}\right) + \left(\frac{0.5-\mu}{\delta}\right) \quad (5)$$

$$\mu = \frac{\bar{X} - \xi}{\lambda} \quad (6)$$

$$SD(x) = \frac{\sigma_x}{\lambda} \quad (7)$$

where:
$SD(x)$ – modified standard deviation,
$\bar{X}$ – arithmetic mean of the plot diameters and heights,
σ_x – plot diameter and height standard deviations.

The value of the location parameter ξ was constrained to equal minimum diameter and minimum height minus 1.34 and 1.15, respectively. These values were derived from the extreme value distribution (Ogana et al. 2018). The scale parameter λ was taken as maximum diameter and maximum height.

Conditional maximum likelihood. Johnson (1949a), Schreuder and Hafley (1977) and

Gorgoso et al. (2012) used the CML estimation method for the shape parameters γ and δ of Johnson's S_B probability density function with predetermined values of location ξ and scale λ parameters. The values of the parameters were obtained with Eq. 8–12:

$$\delta = \frac{1}{S_i} \tag{8}$$

$$\gamma = \frac{-\overline{f_i}}{S_i} \tag{9}$$

$$\overline{f_i} = \sum_{i=1}^{n} \frac{f_i}{n} \tag{10}$$

$$S_i^2 = \frac{\sum_{i=1}^{n}\left(f_i - \overline{f_i}\right)^2}{n} \tag{11}$$

$$f_i = \ln\left(\frac{x_i - \xi}{\xi + \lambda - x_i}\right) \tag{12}$$

where:

S_i – standard deviation,

f_i – transformed variate.

The same procedure with moments was used for λ and ξ.

Mode method. This method was developed by Hafley and Buford (1985) and recently used by Zhang et al. (2003) and Gorgoso-Varela and Rojo-Alboreca (2014). The parameters of the marginal S_B distributions of diameter and height were estimated with the Eqs 13 and 14:

$$\delta = \frac{\lambda}{4\sigma_x} \tag{13}$$

$$\gamma = \frac{2x_m - 2\xi - \lambda}{\lambda\delta} - \delta\ln\left(\frac{x_m - \xi}{\lambda + \xi - x_m}\right) \tag{14}$$

where:

x_m – mode of the random variables *x*, i.e. tree diameters and heights.

The values of the location parameter ξ and the scale parameter λ were also predetermined using the same procedure as with moments.

Knoebel and Burkhart method. This method was developed by Knoebel and Burkhart (1991), and recently used by Gorgoso-Varela and Rojo-Alboreca (2014) and Ogana et al. (2017), below as Eqs 15–18:

$$\gamma = -\delta \times \ln\left(\frac{X_{50} - \xi}{\xi + \lambda - X_{50}}\right) \tag{15}$$

$$\delta = \frac{Z_{95}}{\ln\left(\frac{X_{95} - \xi}{\xi + \lambda - X_{95}}\right) - \ln\left(\frac{X_{50} - \xi}{\xi + \lambda - X_{50}}\right)} \tag{16}$$

$$\xi = X_{\min} - 1.3 \tag{17}$$

$$\lambda = X_{\max} - \xi + 3.8 \tag{18}$$

where:

X_{50}, X_{95} – estimates of the 50th, 95th percentiles of the observed diameter and height data distribution,

Z_{95} = 1.645; standard normal value corresponding to the cumulative percentile of 95%,

$X_{\min}$, $X_{\max}$ – minimum, maximum diameters and heights.

The distribution was fitted using SAS/STAT™ software (Version 9.1.3, 2003).

The performance of the four fitting methods of the S_{BB} distribution model was evaluated by *H* (m) and volume (*V*, m^3) prediction. Individual tree volume equation was developed for the *E. camaldulensis* using *D* and *H*, as Eq. 19:

$$\ln V = -9.349 + 2.072\ln D + 0.891\ln H \tag{19}$$

Following the procedure of Li et al. (2002), observed tree diameters and heights were used in the volume equation to compute the observed tree volume. The predicted tree volumes were determined from the observed diameters and predicted heights by the S_{BB} *H-D* median regression model using the four fitting methods. Relative prediction bias was computed as the percentage of bias over the predicted tree volume. Root mean square error (RMSE) was also computed. Furthermore, the paired sample *t*-test was used to test for significance of differences between observed and predicted tree height, and observed and predicted tree volume at 5% level for each of the 90 plots.

RESULTS

The statistics of the estimated parameters are presented in Table 2. The mean, minimum, maximum and standard deviation is shown in the table. The CML, moments and mode methods had the same values for the location and scale parameters. The location parameter was constrained to minimum diameter minus 1.34 and minimum height minus 1.15 for the marginal diameter and height distributions, respectively. The scale, i.e. lambda, was taken as maximum diameter and maxi-

Table 2. Statistics of the estimated parameters of Johnson's S_{BB} for the different methods

Method		ξ_d	λ_d	γ_d	δ_d	ξ_h	λ_h	γ_h	δ_h	ρ	θ	φ
CML	mean	2.483	25.951	0.918	0.969	3.458	26.194	0.770	0.988	0.678	0.936	0.664
	maximum	14.06	47.428	1.879	1.567	9.25	39.60	1.745	1.359	0.888	2.028	0.943
	minimum	0.66	8.594	0.007	0.595	0.95	9.40	0.086	0.683	0.104	0.395	0.127
	SD	1.568	8.845	0.444	0.175	1.471	6.962	0.341	0.150	0.138	0.364	0.152
Moments	mean	2.483	25.951	0.958	0.936	3.458	26.194	0.806	0.987	0.653	0.901	0.619
	maximum	14.06	47.428	1.812	1.597	9.25	39.60	1.783	1.385	0.983	2.168	1.078
	minimum	0.66	8.594	0.014	0.465	0.95	9.40	0.028	0.547	0.099	0.334	0.128
	SD	1.568	8.845	0.462	0.197	1.471	6.962	0.353	0.179	0.139	0.369	0.158
KB	mean	2.523	27.227	1.186	1.078	3.308	26.686	0.844	1.052	0.769	1.487	0.828
	maximum	14.1	48.708	2.726	2.543	9.10	38.50	2.449	2.147	2.098	13.292	4.211
	minimum	0.70	10.989	–0.046	0.506	0.81	11.20	–0.009	0.442	0.104	0.316	0.118
	SD	1.569	8.655	0.617	0.371	1.471	6.348	0.416	0.319	0.368	1.992	0.581
Mode	mean	2.483	25.951	1.684	1.323	3.458	26.194	1.489	1.309	1.481	9,521.8*	1.513
	maximum	14.06	47.428	4.516	2.092	9.25	39.60	4.248	1.826	5.103*	95,9742.8*	6.136
	minimum	0.66	8.594	–0.556	0.789	0.95	9.40	–0.349	0.894	–0.365	0.109	–0.467
	SD	1.568	8.845	1.078	0.252	1.471	6.962	0.990	0.195	0.910	94,293.5*	1.021

CML – conditional maximum likelihood, KB – Knoebel and Burkhart, SD – standard deviation, ξ – location parameter, λ – scale parameter, γ – asymmetry parameter, δ – kurtosis parameter, d in subscript – diameter, h in subscript – height, ρ – dependence measure between Z_d and Z_h (for details see Eq. 2), *unusually large values, θ, φ – for details see Eq. 3

mum height. The usual constraint was applied for KB method. The φ and rho ρ parameters which determine the regression curve and slope of the S_{BB} median *H-D* model, respectively, had values greater than zero for CML, moments and KB methods. This is usually assumed for a diameter and height model. These values were extremely large for the mode method.

The estimated parameters from the four methods were substituted in the S_{BB} *H-D* model. This was used to predict the individual tree height for a given diameter wherein tree volumes were computed. The results for the paired sample *t*-test comparison between the predicted height and observed height, and predicted volume and observed volume for the four methods are presented in Tables 3–6. Sample plots with significant difference were indicated with asterisk. The result for the S_{BB} fitted with CML showed that there was no significant difference between the observed and predicted heights and between the observed and predicted tree volumes for the 90 plots at 5% level (Table 3). This indicated 100% similarity. In the case of the S_{BB} fitted with moments, the results showed that 89 (98.9%) of the 90 plots were not significantly different from the observed height. While the result for the predicted volume indicated that 85 (94.4%) of the 90 plots were not significantly different from the observed volumes at 5% level (Table 4).

The predicted height by the S_{BB} fitted with KB method was not significantly different from the observed height in 80 (89.9%) of the 90 plots at 5% level. Also, the predicted volume was not significantly different from the observed volume in 68 (75.6%) of the 90 plots (Table 5).

There was a considerable decrease in the performance of the S_{BB} model when the method of mode was used to fit the marginal distribution of diameters and heights (Table 6). The result showed that there was no significant difference between the observed and predicted height in only 29 (32.2%) of the 90 plots. The predicted and observed volumes were not significantly different in 53 (58.9%) of the 90 plots at 5% level. No height prediction was observed in plot 66 with the mode method. There was no mode value in the diameter distribution of the plot; as such, the gamma parameter (γ_d) was not estimated for the marginal diameter distribution.

The predicted tree height from S_{BB} *H-D* fitted with the four methods and the observed tree height are presented in Fig. 1 (smooth curves). One representative sample plot (Plot 78) was displayed. Tree height prediction from CML and moments was more sensitive than from the KB and mode methods. The S_{BB} *H-D* fitted with mode overestimated tree height in the larger diameters.

The average relative bias (%) of tree height and volume predictions from the S_{BB} distribution fitted with CML, moments, KB and mode for the 90 plots are illustrated in Figs 2a, b, respectively. The height prediction based on CML and moments had smaller percentage bias compared to the height

Table 3. Paired sample *t*-test comparison of the predicted tree height and volume for S_{BB} fitted with conditional maximum likelihood

Plot	*t*-Value		*df*	Plot	*t*-Value		*df*	Plot	*t*-Value		*df*
	height	volume			height	volume			height	volume	
1	0.37	0.95	33	31	0.39	0.29	46	61	0.06	0.57	32
2	0.70	0.85	37	32	0.90	0.01	44	62	0.55	0.60	33
3	0.47	0.14	40	33	0.69	0.34	43	63	0.50	0.84	29
4	0.73	0.54	33	34	1.00	1.15	64	64	0.64	0.31	34
5	1.02	1.28	40	35	0.98	0.67	43	65	0.69	0.04	34
6	0.51	0.26	42	36	0.73	1.08	27	66	0.68	0.49	30
7	0.40	0.18	44	37	0.39	0.61	57	67	0.38	0.46	31
8	0.63	0.45	51	38	1.37	0.19	60	68	0.40	0.64	56
9	1.09	0.51	61	39	1.00	0.48	48	69	0.48	0.95	54
10	0.71	0.90	45	40	0.35	0.10	40	70	0.69	0.70	55
11	0.33	0.03	54	41	0.58	0.12	38	71	0.55	0.91	54
12	0.86	0.17	30	42	1.25	0.20	61	72	0.23	0.35	31
13	0.27	0.19	70	43	0.39	0.20	48	73	0.51	0.53	55
14	1.00	0.82	82	44	1.12	0.08	60	74	0.33	0.86	44
15	0.96	0.80	38	45	0.81	0.15	43	75	0.67	0.63	57
16	1.28	0.97	70	46	0.48	0.63	52	76	0.01	0.84	34
17	1.13	0.41	79	47	0.02	0.81	40	77	0.40	0.37	54
18	0.81	1.84	54	48	1.06	0.49	51	78	0.73	1.39	57
19	0.32	0.29	34	49	0.51	0.35	41	79	0.47	0.34	52
20	0.70	0.85	72	50	0.55	0.97	33	80	0.57	0.30	53
21	0.40	0.45	49	51	0.93	0.82	47	81	0.97	0.29	57
22	0.90	1.10	44	52	0.91	0.49	30	82	0.42	0.13	54
23	0.97	1.60	39	53	0.86	0.73	27	83	0.35	0.12	53
24	1.19	1.55	52	54	1.81	1.89	37	84	0.67	0.23	31
25	0.93	0.80	48	55	0.55	0.35	29	85	0.19	0.86	37
26	0.62	0.85	40	56	1.01	0.82	29	86	0.57	0.26	71
27	0.74	0.84	64	57	0.56	0.29	28	87	0.82	0.13	54
28	0.69	0.17	41	58	0.31	0.41	36	88	0.92	0.51	60
29	1.03	0.43	50	59	0.56	0.46	35	89	0.49	0.58	47
30	0.69	0.46	38	60	0.56	0.12	34	90	0.81	1.00	50

df – degree of freedom

prediction based on KB and mode methods in all plots. Most of the values lied within a narrow band of < ± 5% for CML- and moment-based methods. The overall average relative biases (%) of tree height across the 90 plots were 3.23, 3.01, 6.19 and 8.29 with corresponding RMSE values of 3.66, 3.70, 4.17 and 6.92 for CML, moments, KB and mode, respectively. Also, the volume prediction based on CML, moments and mode had smaller percentage bias compared to the volume prediction based on KB method in all plots. Most of the values lied within the range of –10 to 10 for CML-, moment- and mode-based methods. The volume prediction biases (%) from the KB method were relatively large in most of the plots. The overall average relative biases (%) of tree volume across the 90 plots were 3.34, 3.12, 5.72 and 1.32 with corresponding RMSE values of 0.08, 0.08, 0.10 and 0.03 for CML, moments, KB and mode, respectively. However, most of the volume predictions from the mode method were significantly different from the observed tree volume at 5% level as reported in Table 5. The overall performance of the four methods of Johnson's S_{BB} distribution for predicting tree height and volume could be summarized as: CML ≈ moments > KB > mode.

DISCUSSION

This study has evaluated the performance of Johnson's S_{BB} distribution fitted with CML, moments, KB and mode for predicting tree height and tree volume. The results obtained were not un-

Table 4. Paired sample *t*-test comparison of the predicted tree height and volume for S_{BB} fitted with moments

Plot	*t*-Value		*df*	Plot	*t*-Value		*df*	Plot	*t*-Value		*df*
	height	volume			height	volume			height	volume	
1	0.01	1.27	33	31	0.37	0.01	46	61	1.61	1.95	32
2	0.67	0.92	37	32	0.95	0.10	44	62	0.70	0.63	33
3	0.59	0.73	40	33	0.71	0.25	43	63	0.25	1.07	29
4	1.23	0.99	33	34	1.64	2.45*	64	64	1.02	1.17	34
5	1.22	1.53	40	35	1.06	1.28	43	65	0.18	0.84	34
6	0.44	0.73	42	36	0.31	0.09	27	66	0.65	0.75	30
7	1.04	1.10	44	37	0.65	0.89	57	67	0.33	0.94	31
8	1.35	1.45	51	38	0.10	0.31	60	68	0.43	0.73	56
9	1.62	1.72	61	39	0.38	0.71	48	69	0.52	0.66	54
10	0.94	1.56	45	40	0.70	0.43	40	70	0.21	0.67	55
11	0.33	0.32	54	41	0.85	0.23	38	71	0.50	1.16	54
12	0.88	0.06	30	42	0.46	0.36	61	72	0.44	0.11	31
13	0.11	0.32	70	43	0.47	0.28	48	73	1.24	0.16	55
14	1.95	2.05*	82	44	1.17	0.10	60	74	0.43	0.47	44
15	0.82	0.74	38	45	0.03	0.11	43	75	0.65	0.54	57
16	0.66	1.25	70	46	1.01	1.30	52	76	0.31	0.30	34
17	1.69	1.38	79	47	0.29	1.50	40	77	0.86	0.87	54
18	0.17	1.84	54	48	0.49	0.59	51	78	0.02	0.82	57
19	0.91	1.07	34	49	0.40	0.75	41	79	0.01	0.20	52
20	0.87	0.50	72	50	0.61	1.15	33	80	1.45	1.39	53
21	0.43	0.37	49	51	0.82	0.99	47	81	1.06	0.42	57
22	0.86	1.30	44	52	0.97	1.22	30	82	0.69	0.18	54
23	0.36	0.73	39	53	1.40	1.46	27	83	0.97	0.47	53
24	1.52	1.89	52	54	1.60	2.48*	37	84	0.55	0.05	31
25	0.85	1.00	48	55	2.08*	1.99	29	85	0.31	0.72	37
26	0.08	0.65	40	56	1.57	2.94*	29	86	1.46	0.04	71
27	1.15	1.33	64	57	1.07	0.82	28	87	0.15	1.08	54
28	0.37	0.14	41	58	0.20	2.04*	36	88	0.58	0.47	60
29	0.26	0.45	50	59	0.07	1.35	35	89	0.04	0.40	47
30	0.07	0.20	38	60	1.28	1.66	34	90	0.82	0.89	50

*significance at 5%, *df* – degree of freedom

expected. No significant difference was observed between the predicted and observed tree height, and predicted and observed tree volume for the CML-based method at 5% level. This is an indication of the suitability of the CML-based method for fitting Johnson's S_{BB} distribution compared to other fitting methods considered in this study. The parameter estimates from the CML-based method are reasonable. For example, the values of ϕ and ρ parameters which determine the regression curve and slope of the S_{BB} median *H-D* model are greater than zero. This is usually assumed for a diameter and height model. The CML-based method was applied by Tewari and von Gadow (1999) to establish the S_{BB} median regression relationship between tree diameters and heights. This was used to compute 5- and 95-percentile curves. The authors concluded that the percentiles obtained through the bivariate distribution showed that the variation in height for a given diameter was less pronounced in the larger trees. Schreuder and Hafley (1977) and Omule (1984) also reported good results with the CML method for fitting Johnson's S_{BB} distribution.

The performance of the moments-based method was comparable to the CML. Only 1 of the 90 plots was significant for the predicted tree height; while 5 of the 90 plots were significant for the predicted tree volume at 5% level. The relative average bias of tree heights and volumes was smaller than the CML values. The parameter estimates from the moments-based method are more or less similar to the CML. This means that the moments-based method can be used for fitting bivariate Johnson's S_{BB} distribution in lieu of CML. No published literatures exist on the S_{BB} distribution fitted with

Table 5. Paired sample *t*-test comparison of the predicted tree height and volume for S_{BB} fitted with Knoebel and Burkhart method

Plot	*t*-Value		*df*	Plot	*t*-Value		*df*	Plot	*t*-Value		*df*
	height	volume			height	volume			height	volume	
1	0.61	1.06	33	31	0.29	1.96	46	61	1.96	3.50*	32
2	0.81	1.85	37	32	0.25	0.14	44	62	0.74	0.87	33
3	0.13	0.53	40	33	0.51	0.33	43	63	0.01	1.51	29
4	0.87	1.01	33	34	1.62	1.17	64	64	1.21	0.05	34
5	3.21*	3.81*	40	35	1.18	3.55*	43	65	0.46	0.77	34
6	0.72	1.84	42	36	1.62	1.52	27	66	0.05	2.16*	30
7	0.18	1.74	44	37	0.91	1.33	57	67	1.33	2.92*	31
8	1.43	1.80	51	38	1.54	0.83	60	68	1.33	1.26	56
9	1.87	2.21*	61	39	0.81	0.81	48	69	0.35	0.39	54
10	0.46	0.26	45	40	1.49	0.45	40	70	0.26	0.13	55
11	0.23	0.46	54	41	2.07*	1.96	38	71	0.49	1.72	54
12	0.47	0.88	30	42	0.05	0.68	61	72	1.55	1.04	31
13	2.36*	2.39*	70	43	1.62	0.43	48	73	1.07	0.34	55
14	0.75	1.63	82	44	0.35	0.47	60	74	2.08*	2.68*	44
15	2.21*	3.01*	38	45	0.06	1.53	43	75	0.91	2.20*	57
16	0.64	0.94	70	46	0.98	1.54	52	76	0.24	0.70	34
17	1.14	0.41	79	47	2.2*	0.09	40	77	0.18	0.40	54
18	3.72*	1.99	54	48	0.76	0.96	51	78	1.83	2.79*	57
19	1.59	1.84	34	49	0.68	0.79	41	79	0.17	2.01*	52
20	0.50	2.28*	72	50	0.49	1.29	33	80	2.53*	2.50*	53
21	0.79	1.19	49	51	0.44	0.71	47	81	0.94	0.38	57
22	0.19	2.33*	44	52	1.84	0.29	30	82	1.73	1.61	54
23	0.75	2.38*	39	53	1.06	1.14	27	83	3.48*	1.85	53
24	1.30	1.96	52	54	1.12	2.83*	37	84	0.37	1.48	31
25	0.61	0.83	48	55	1.78	1.93	29	85	1.27	0.45	37
26	0.64	1.12	40	56	1.73	2.91*	29	86	1.46	0.08	71
27	0.62	0.91	64	57	1.05	0.33	28	87	1.91	2.67*	54
28	0.43	0.62	41	58	0.05	2.56*	36	88	0.60	1.81	60
29	1.29	1.36	50	59	1.80	1.25	35	89	1.96	2.29*	47
30	0.37	1.62	38	60	2.19*	1.20	34	90	1.89	2.22*	50

*significance at 5%, *df* – degree of freedom

moments to the best of my knowledge. However, there are a number of studies on univariate Johnson's S_B distribution fitted with moments. For example, Gorgoso-Varela and Rojo-Alboreca (2014) compared CML, moments, mode and KB. The CML- and moments-based methods provided the best fit for Johnson's S_B distribution, while the mode-based method had the worst fit. Similar results were reported by Ogana et al. (2017) for the diameter distribution of *Gmelina arborea* Roxburgh.

There was a reduction in the predictive ability of the S_{BB} distribution fitted with KB- and mode-based methods. Their predictions were characterised by under- and overestimation of the height and volume, with significant differences observed in most of the plots, especially for the mode-based method. The results obtained in Li et al. (2002) further confirmed that the KB-based method might not be suitable for fitting the S_{BB} distribution to the joint distribution of diameters and heights. The authors evaluated the performance of the GBD-2 and S_{BB} distributions in terms of height and volume predictions. Their results showed that the relative percentage bias of the GBD-2 was three times smaller than the S_{BB} distribution fitted with KB method. Also, Mønness (2015) compared the S_{BB}, bivariate power-normal distributions and hyperbolic height model based on their ability to generate height curves. Though both distributions performed well but the hyperbolic height curve had the smallest height deviation. The author used maximum like-

Table 6. Paired sample *t*-test comparison of the predicted tree height and volume for S_{BB} fitted with mode

Plot	*t*-Value		*df*	Plot	*t*-Value		*df*	Plot	*t*-Value		*df*
	height	volume			height	volume			height	volume	
1	1.84	3.29*	33	31	0.27	0.40	46	61	2.86*	3.41*	32
2	7.42*	1.86	37	32	2.32*	0.45	44	62	6.26*	0.78	33
3	1.95	0.22	40	33	4.31*	2.55*	43	63	6.92*	2.28*	29
4	2.69*	1.00	33	34	4.64*	1.80	64	64	2.68*	3.08*	34
5	6.74*	1.02	40	35	4.26*	0.07	43	65	4.75*	2.13*	34
6	8.06*	0.28	42	36	10.34*	0.34	27	66†	–	4.34*	30
7	6.33*	0.29	44	37	2.33*	0.69	57	67	9.94*	4.94*	31
8	1.67	3.00*	51	38	10.85*	2.46*	60	68	7.42*	3.29*	56
9	1.39	1.12	61	39	9.41*	1.77	48	69	0.46	2.12*	54
10	2.93*	1.37	45	40	0.75	0.07	40	70	2.32*	2.58*	55
11	4.4*	1.25	54	41	5.78*	0.82	38	71	5.89*	3.41*	54
12	1.49	0.73	30	42	0.71	0.77	61	72	2.06*	0.81	31
13	0.85	3.15*	70	43	0.24	2.92*	48	73	9.35*	1.43	55
14	0.70	0.51	82	44	4.28*	1.98	60	74	2.83*	1.27	44
15	6.26*	1.44	38	45	3.41*	2.25*	43	75	7.75*	0.81	57
16	1.40	3.96*	70	46	5.47*	2.15*	52	76	7.11*	0.83	34
17	4.67*	0.87	79	47	1.46	4.34*	40	77	7.51*	0.43	54
18	9.92*	2.02*	54	48	0.39	2.44*	51	78	0.93	1.16	57
19	3.48*	1.33	34	49	0.84	2.45*	41	79	0.39	2.64*	52
20	9.6*	0.85	72	50	3.92*	3.56*	33	80	0.91	0.25	53
21	6.35*	1.29	49	51	9.08*	1.48	47	81	2.47*	0.31	57
22	4.65*	3.70*	44	52	1.36	2.58*	30	82	5.90*	0.32	54
23	4.27*	3.75*	39	53	4.44*	2.19*	27	83	0.87	1.12	53
24	5.02*	1.82	52	54	3.86*	2.29*	37	84	3.11*	0.40*	31
25	0.66	0.83	48	55	0.12	1.15	29	85	2.99*	1.28	37
26	0.12	1.69	40	56	0.40	1.03	29	86	1.96	0.73	71
27	6.81*	1.12	64	57	0.40	0.21	28	87	4.16*	0.47	54
28	5.58*	1.84	41	58	2.13*	3.02*	36	88	4.52*	0.23	60
29	7.49*	2.31*	50	59	4.06*	2.73*	35	89	9.49*	2.51*	47
30	7.11*	2.71*	38	60	3.32*	2.51*	34	90	0.92	1.75	50

*significance at 5%, *df* – degree of freedom, †no fit was observed

lihood estimation to obtain the parameters of the S_{BB} distribution. Perhaps, if the CML- or moments-based methods were used, this bias would have been minimized. The poor performance of the mode-based method may be because both the mode and minimum diameter and height values were more or less similar in most of the plots. Usually, when this occurs, the mode-based method will yield poor fits (Zhang et al. 2003). For this reason, Hafley and Buford (1985) suggested that the mode method should not be used where the mode of the distribution lies at diameter and height extremes. This is a major restriction to the used mode-based method to fit Johnson's distribution.

Fitting the S_{BB} distribution usually requires that ξ_d, λ_d and ξ_h, λ_h values be predetermined. One common practice with the S_{BB} *H-D* model is to assign ξ_d and ξ_h (location parameters) of the diameter and height distributions to be zero and 1.3 m, respectively; that is, at 1.3 m tree height, the DBH should be zero. When this assumption was used in this study, no improvement in height prediction was observed.

In conclusion, this study has considered four commonly used estimation methods of fitting the bivariate S_{BB} distribution. The results vary greatly across the methods, with CML and moments having the best performance. The tree height and volume predictions of Johnson's S_{BB} distribution fitted with CML and moments are reasonable; and are more or less the same with observed tree height and volume. Thus, the CML- and moments-based

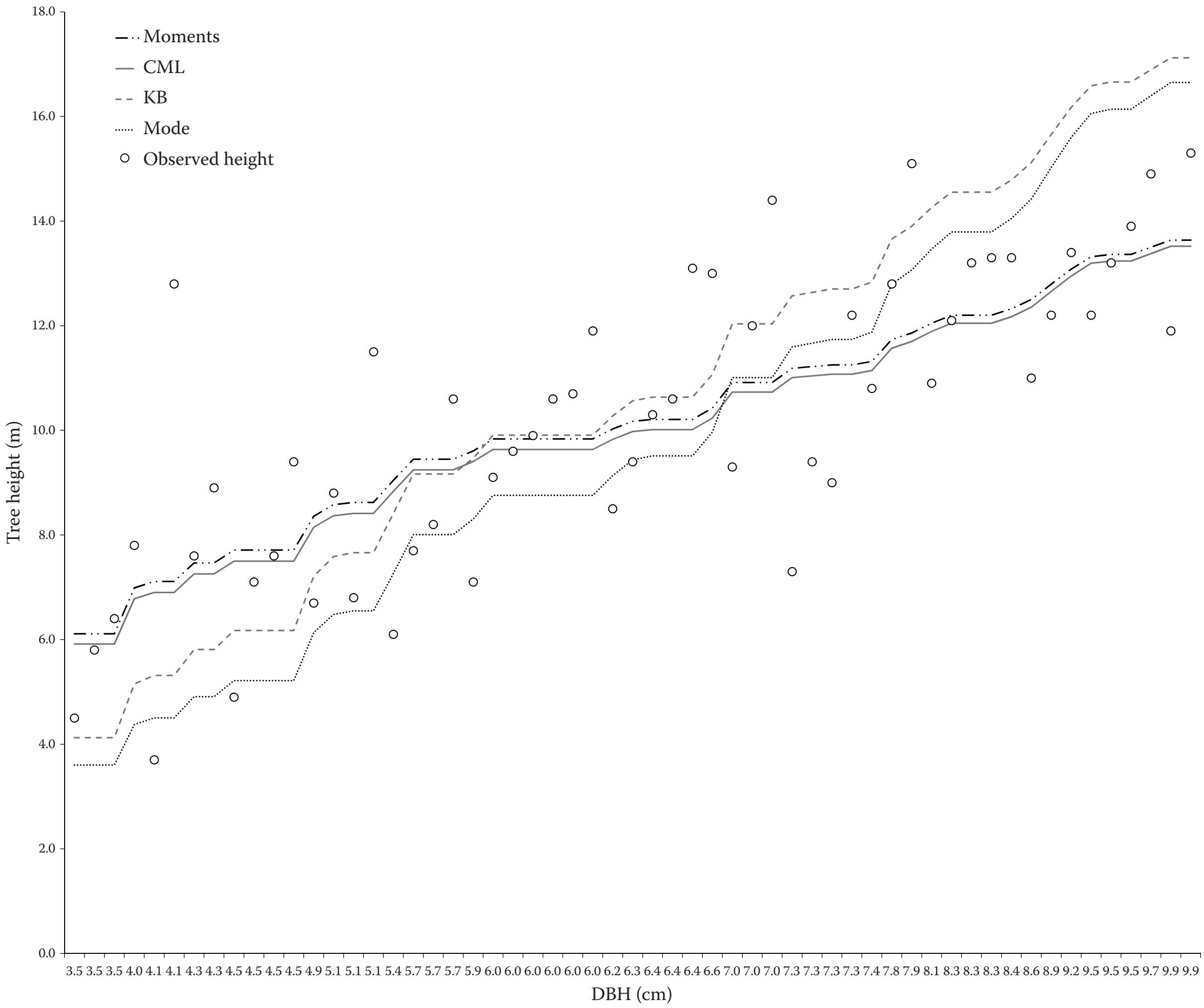

Fig. 1. Tree height prediction from S_{BB} *H-D* fitted with moments, conditional maximum likelihood (CML), Knoebel and Burkhart (KB) and mode methods for Plot 78

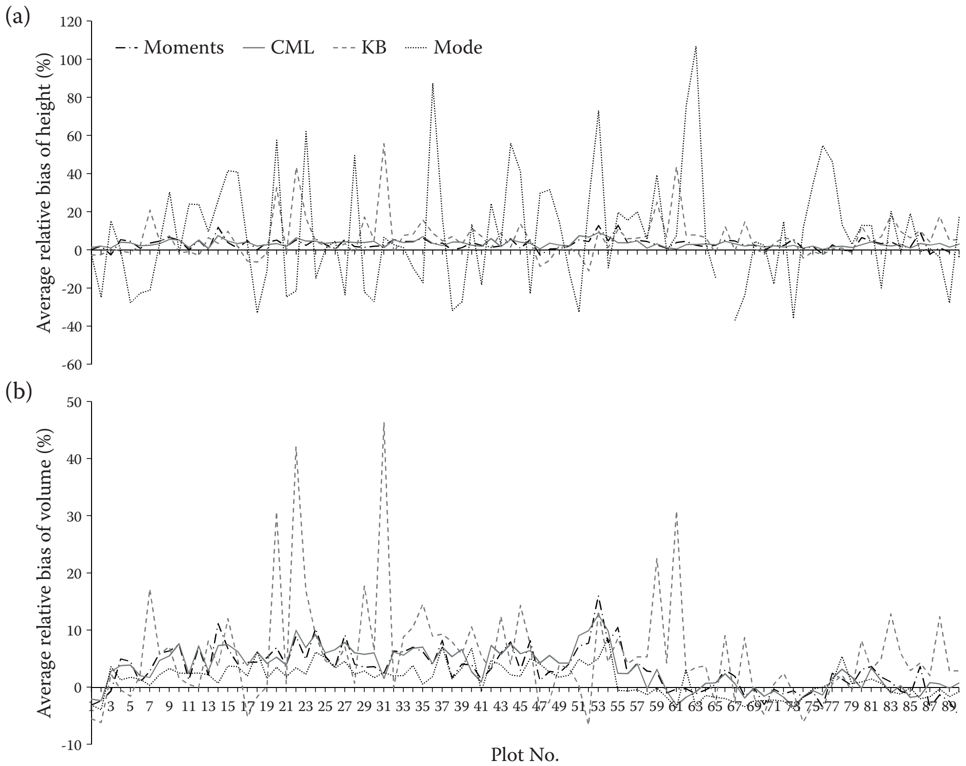

Fig. 2. Average relative bias of height (a), volume (b) prediction from Johnson's S_{BB} distribution fitted with moments, conditional maximum likelihood (CML), Knoebel and Burkhart (KB) and mode methods for all 90 plots

methods are recommended for fitting bivariate Johnson's S_{BB} distribution for efficient timber management.

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