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Multivariate analysis for assessment of the tree populations based on dendrometric data with an example of similarity among Norway spruce subpopulations

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Abstract

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The new method for evaluation of tree populations presented here is based on a correlation analysis within a set of dendrometric variables. The correlation analysis is carried out for each population separately. The method evaluates differences between resulting correlation matrices. These distances can be used by hierarchical cluster analysis (unweighted pair-group average) or by ordination analysis (non-metric multidimensional scaling – NMS). Test data were obtained in 10 research plots in the area of Medvědí Mt., Šumava National Park. Plots are located in Norway spruce [*Picea abies* (Linnaeus) H. Karsten] climax forests. The results enable ecological interpretation of both classification and NMS. The populations (subpopulations) differ in their origin (spontaneous succession or partial planting) and environmental conditions (extreme environment near the mountain summit versus water-logged soils). These differences were reflected in results of the classification and ordination of the spruce (sub)populations.

Keywords: dendrometry; hierarchical classification; ordination; *Picea abies*

Mensuration represents the basic step of data collection in forest (van Laar, Akça 2007). These methods are primarily designed to evaluate development of standing volume over time, and they have a high predictive value with respect to ecology of the stand (Pretzsch 2009).

The population of trees is usually evaluated using a set of variables such as DBH, tree height, crown projection area, crown length and height of the crown base. This describes all individuals belonging to the population. Classically, the data are evaluated using basic descriptive statistics (such as mean, median, standard deviation). Due to the fact that variables can be more or less related, it is advisable to use also correlation or regression statistics for the description of the population (West 2009). Known relations are also used for deriving frequently used allometric relationships as described by Fralish (1988), Černý (1990), Osawa and Allen (1993), Bartelink (1997, 1998), Sumida et al. (1997), and Blujdea et al. (2012). Typical is the relation between tree height and diameter, where a set of models exists (Huxley, Teissier 1936; Näslund 1936; Meyer 1940; Bertalanffy 1957; Curtis 1967; Bates, Watts 1980; Wykoff et al. 1982; Buford 1986; Huang et al. 2000; Staudhammer, LeMay 2000). Such relationships are frequently non-linear. Today there are also entire databases of these relationships (Jenkins et al. 2003; Somogyi et al. 2008; Henry et al. 2013). For some tree species, the relationship between dendrometric variables within a large part of their distribution area is known (Wirth et al. 2004; Dimitris et al. 2005).

Although the data measured on a set of trees has a multidimensional character, this fact is generally overlooked and, for example, a book by Robinson and Hamann (2011) did not work with this fact. Therefore, we have prepared a methodology presented here that seeks to bridge this gap.

The basic goal of this paper is to introduce a new multidimensional method (combination of methods) for evaluation of a set of tree populations (subpopulations) on the basis of dendrometric data. The method will be applied on an example of ten subpopulations of *Picea abies* (Linnaeus) H. Karsten in the Šumava Mts. (Bohemian Forest, Czech Republic). This application leads to secondary questions: Are there any relations between tree subpopulation features and local environmental conditions? Are comparable results based on dendrometric data and on species composition of plant communities?

MATERIAL AND METHODS

Method description. Conventional mensurational data that are gathered during the investigation within a number of populations or subpopulations of a single tree species can be processed using the procedure introduced below. Basic units can be represented, for example, by research plots where all tree individuals are measured.

Since most dendrometric variables do not have a normal distribution, which is already apparent from the nature of these variables (the values cannot be negative, the upper limit is not specified, the distribution may be asymmetric), it is in practical terms more suitable to take the logarithm of these variables rather than proper variables alone.

The relationship of the individual variables was assessed using Pearson's correlation coefficient (r). A set of correlation coefficients (correlation matrix) is used in many multidimensional statistical analyses, principal component analysis (PCA) can be mentioned first of all (Legendre, Legendre 2012). Statistical significance of the difference in the respective correlation coefficients r_1 and r_2 for two compared plots is tested by the variable U (Anděl 1985), as Eq. 1:

$$U = \frac{z_1 - z_2}{\sqrt{\frac{1}{n_1 - 3} + \frac{1}{n_2 - 3}}} \tag{1}$$

where:

$z_1 = \frac{1}{2} \ln \frac{1 + r_1}{1 - r_1}$

z_2 – calculated in a similar way like z_1,

n_1, n_2 – number of objects (trees) in sets 1 and 2.

Because the variable U has an approximately normal distribution N(0, 1), it can be tested against the critical values of a normal distribution. A pair of plots, where the respective correlation coefficients between the variables describing the size of the trees (or between the logarithms of these variables) were significantly different, is considered to be plots with different tree growth features (even with different habit). Critical values for U are therefore equal to 1.960 at a significance level of $\alpha = \pm 0.05$, and 2.576 at a significance level of $\alpha = \pm 0.01$ (two-sided test). The absolute value $|U|$ can be considered to be a measure of dissimilarity between the two evaluated populations. This has been applied in the paper by Bednařík and Matějka (2014).

Given the above, it is possible to create a matrix over m populations (plots), as Eq. 2:

$$\boldsymbol{U} = \begin{bmatrix} 0 & U_{1,2}^2 & \dots & U_{1,m}^2 \\ U_{2,1}^2 & 0 & \dots & U_{2,m}^2 \\ \dots & \dots & \dots & \dots \\ U_{m,1}^2 & U_{m,2}^2 & \dots & 0 \end{bmatrix} \tag{2}$$

This matrix can be used in hierarchical agglomerative clustering (Meloun et al. 2012; StatSoft, Inc. 2013).

Regarding the fact that correlations can be calculated for a set of n variables (e.g. variables v and w; $1 \le v, w \le n$), a set of appropriate differences ($U_{(p, q), (v, w)}$) exists. These differences can be summed for the pair of plots p and q as the sum of squares (Eq. 3):

$$X_{p,q}^2 = \sum_{v=2} \sum_{w=1} U_{(p,q),(v,w)}^2 \tag{3}$$

Regarding the normal distribution of U, the variable X^2 shows χ^2-distribution with $(n \times (n - 1)/2) - 1$ degrees of freedom. Also $X_{p,q}^2$ can be considered as the distance (dissimilarity) between the compared populations p and q ($1 \le p, q \le m$) and it can be used to classify the entire group of populations or to ordinate by a method as non-metric multidimensional scaling – NMS (McCune, Grace 2002).

The procedure of unweighted pair-group average (UPGA) was used as a classification. It was calculated in the STATISTICA software (Version 8, 2007). The NMS ordination was carried out in the PC-ORD software (Version 6.0, 2011).

The results obtained from dendrometric data are compared with the classification of vegetation (phytocoenological relevés) from the same plots (Bednařík, Matějka 2011). The phytocoenological relevés were classified using Ward's method. The resulting dendrogram was compared with NMS ordination based on dendrometric data within DBreleve software, ANOVA, with Monte Carlo significance computing (Matějka 2017).

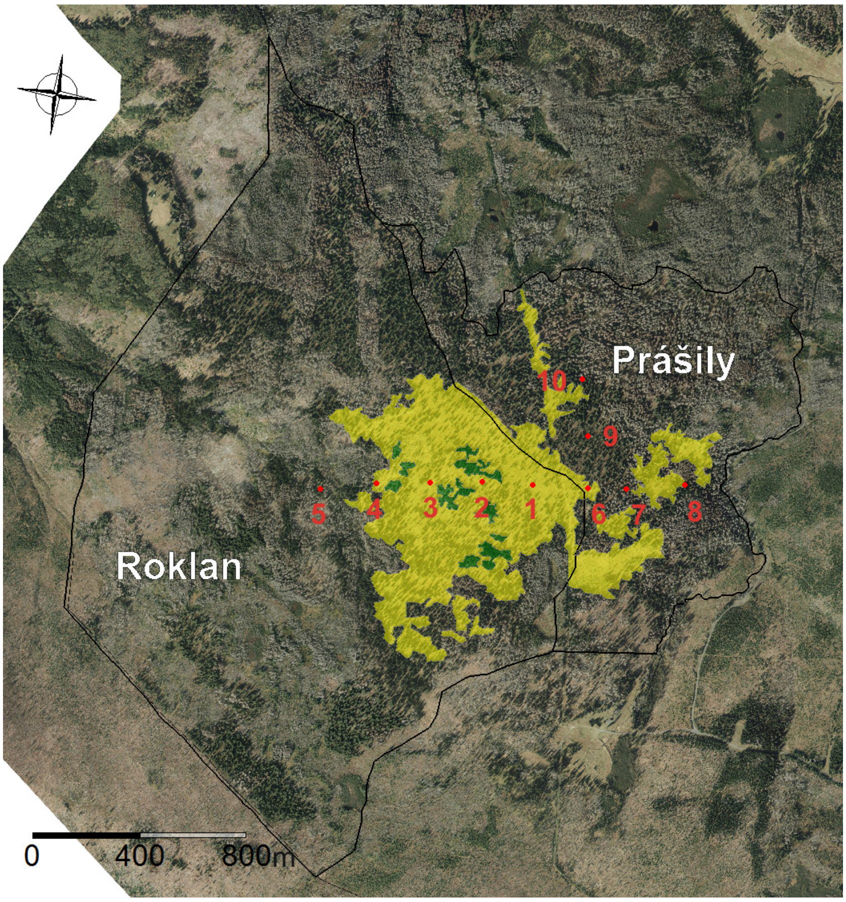

Fig. 1. Distribution of research plots B1 to B10 in the territory of the Medvědí Mt. massif in the Šumava National Park (so-called Jewish Forest) on the basis of the orthophoto map from 2011. A region with a very sparse forest is marked in the central part of the area around the top of Medvědí Mt. (in yellow). This area was defined in the orthophoto as a site where the distance between neighbouring trees is generally greater than 7.5 m (approximately half of the average height of trees in the area of interest)

Example of application of the procedure presented – study site and data analysis. In total 10 square research plots of 0.25 ha in size (50 × 50 m) being roughly 200 m apart were established on Medvědí Mt. (within an actually non-managed area in the central part of the Šumava National Park). The plots are situated in lines from the top of Medvědí Mt. to the west (5 plots), east (3 plots) and north (2 plots) (Fig. 1). The bedrock is dominated by the Moldanubian rock series (mostly schist gneiss, paragneiss, and composite gneiss). The soil cover consists mostly of haplic and entic Podzols, Histosols and marginally also Gleysols. The mean annual temperature in the area ranges between 2 and 3°C and the mean annual precipitation exceeds 1,200 mm. The potential vegetation is classified as *Calamagrostio villosae-Piceetum*, *Mastigobryo-Piceetum* and *Sphagno-Piceetum* (Bednařík et al. 2014).

The canopy was recorded using the FieldMap technology (http://www.fieldmap.cz). The set of determined parameters contains tree height (h), DBH, crown projection area, height of crown base (h_{cr}), crown length and h_{cr}/h ratio. Basic tree features according to the plots are given in Table 1. A detailed

Table 1. Basic dendrometric features of the Norway spruce subpopulations on plots B1 to B10 (Bednařík, Matějka 2011, 2014; Bednařík et al. 2014)

	Plot																			
	B1		B2		B3		B4		B5		B6		B7		B8		B9		B10	
	mean	SD	mean	SD	mean	SD	mean	SD	mean	SD	mean	SD	mean	SD	mean	SD	mean	SD	mean	SD
N	8		38		46		31		175		45		37		58		102		48	
A (m^2)	13.7	13.7	26.0	23.2	14.2	19.8	13.9	10.3	8.7	7.3	15.9	15.4	23.4	17.7	21.4	13.8	10.1	14.6	33.8	16.7
DBH (cm)	24.0	21.9	32.6	24.8	18.9	20.7	33.2	15.9	21.8	14.6	24.7	20.3	39.1	19.3	39.1	14.7	14.0	14.8	48.0	13.8
h (m)	9.2	6.4	12.0	7.1	7.9	6.7	18.3	7.7	12.0	6.6	9.5	4.1	17.7	6.5	19.7	4.5	6.7	4.4	20.3	3.9
h_{cr} (m)	0.2	0.1	0.8	0.9	0.5	0.4	3.8	3.3	2.3	2.3	0.3	0.1	1.8	1.3	2.8	2.9	0.6	0.3	2.2	1.8
l (m)	9.0	6.4	11.2	7.3	7.3	6.7	14.5	7.2	9.8	6.2	9.2	4.2	15.9	6.3	16.9	5.1	6.1	4.4	18.1	4.5
h_{cr}/h (%)	3.3	2.6	11.2	16.1	13.7	15.5	22.6	15.3	21.6	16.8	4.0	2.8	13.2	13.6	14.6	14.4	14.0	13.1	12.3	11.6

N – number of trees, A – crown projection area, h – tree height, h_{cr} – height of crown base, l – crown length, SD – standard deviation description of the study area, methods and some results were introduced by Bednařík and Matějka (2011, 2014) and Bednařík et al. (2014).

Due to asymmetry in the distribution of measured characteristics, the logarithmic transformation of these values was performed before further calculations. The difference in the habit of Norway spruce on the two plots was evaluated as a difference in the respective correlation coefficients r_1 and r_2 and/or as a difference between two sets of correlation coefficients (each set for all pairs of measured variables).

RESULTS

Examples of scatterplots for three selected variable combinations are drawn in Fig. 2, where it is obvious that the logarithm-transformation of data leads to sufficient linearization of the relationships between variables.

When evaluating the relationship between the height of the tree and its DBH (Fig. 3a), the most different populations of Norway spruce were found within the group of plots B8–B10 in the northeastern part of the study area where the forests were less affected by historical interventions (deforestation and subsequent grazing). Spruce planting was particularly important in plot B9. The structure of the stands in this group of plots corresponds to production forests with thinning in the mountain zone. Tree canopy cover ranged between 34 and 56%.

The remaining plots are in the area with very sparse tree canopy, the forests of which have mainly resulted from spontaneous succession. For B5 plot on heavily waterlogged soil, a different character was revealed. It has never been affected by greater artificial deforestation and grazing by cattle, however it is not a typical commercial forest.

If the pairwise correlations between all 6 variables are used for classification, the result is slightly different (Fig. 3b). The most different populations of Norway spruce were found in plots B8 and B10, which represent older, more or less canopy-closed stands. These are followed by plots B4 and B5, which were less (B4) or least (B5) influenced by the historical intervention; they seemed to be a remnant of the original vegetation on sites with an elevated water table level. Plot B9 represents a heavily influenced stand with artificial spruce planting. The remaining plots were affected by cutting and subsequent cattle grazing, located in the extreme environmental conditions around the top of Medvědí Mt. Lower average tree height (7.9 to 16.2 m) corresponds to the local environment. Current canopy cover is only 4 to 30%.

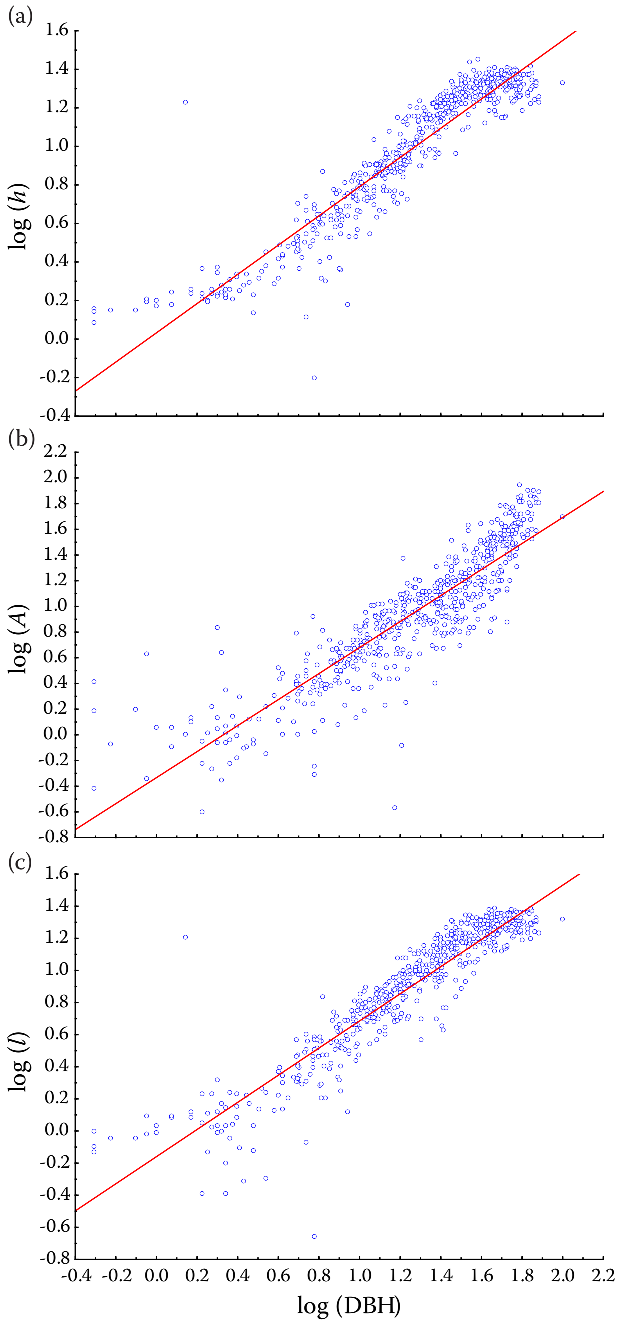

Fig. 2. Examples of individual correlations between the logarithm of DBH and tree height (*h*; $r = 0.9393$, $P < 0.001$) (a), crown area (*A*; $r = 0.8864$, $P < 0.001$) (b), crown length (*l*; $r = 0.9321$, $P < 0.001$) (c). Data of all plots were used

The corresponding NMS ordination (Fig. 4) reveals four basic groups of Norway spruce populations, which are bound to both ordination axes. The first axis distinguishes between populations in areas of disturbed vegetation (cutting, grazing) at extreme positions (the dry shallow soil around the top of Medvědí Mt.), which are located on the left hand side of the ordination space. On the right hand side, we find populations in localities with deeper

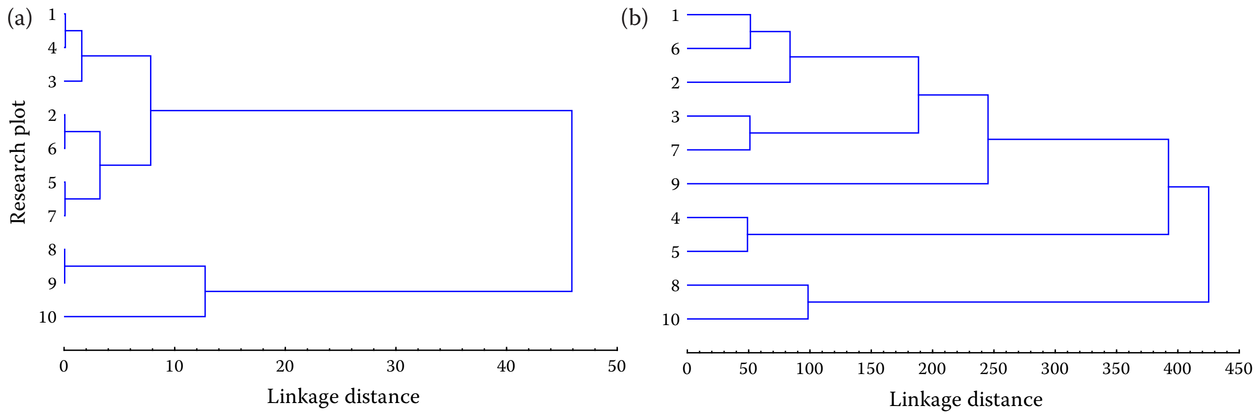

Fig. 3. The unweighted pair-group average classification of populations of *Picea abies* (Linnaeus) H. Karsten on research plots B1 to B10 in the studied area (Šumava National Park), dissimilarity of the population expressed using the variable U^2 calculated on the basis of the correlation coefficients for tree height and DBH (a), X^2 (b)

soils and higher canopy cover. The second axis differentiates populations in areas with waterlogged soils (lower part of the ordination space) and plot B9 at the top of the ordination space, which represents a stand with planting.

The phytocoenological relevés were classified using Ward's method (Fig. 5). The first NMS axis is significantly related to the vegetation classification on the highest classification level (P = 97.3%) or on the two highest levels (P = 95.7%). Main groups of plots according to dendrometric data correspond to the classification of vegetation (Bednařík, Matějka 2011) with the exception of one plot: B4 plot has vegetation similar to the other plots with sparse tree canopy, but Norway spruce dendrometry points to water-logged plots. This plot is situated in a transition zone between opened and closed canopy and the border of the water-logged area is similar.

Variability in the habitus of spruce trees in the investigated area cannot be expressed simply in a single gradient, but there are at least two independent complex characteristics that correspond to the two axes of the ordination space.

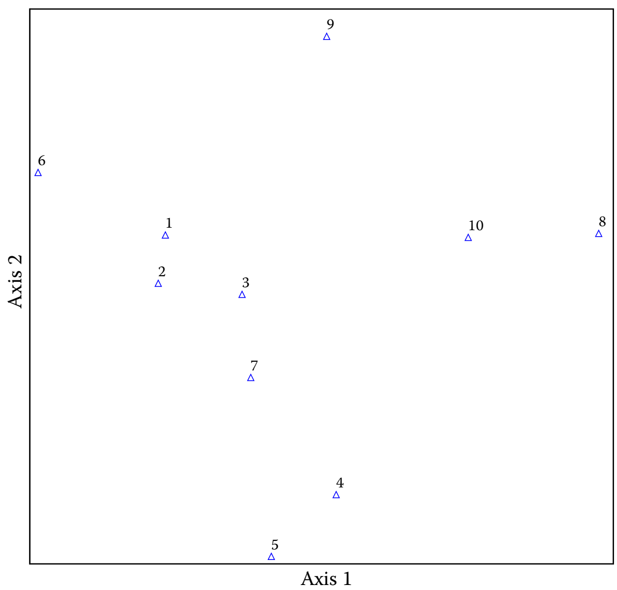

Fig. 4. The non-metric multidimensional scaling ordination of populations of *Picea abies* (Linnaeus) H. Karsten on research plots B1 to B10 in the studied area (Šumava National Park), dissimilarity of the population expressed using the variable X^2

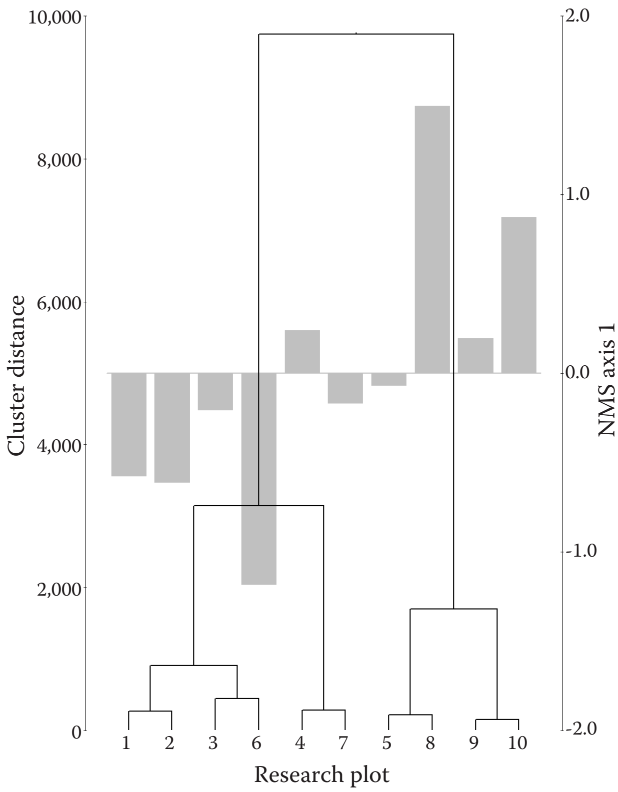

Fig. 5. Classification of vegetation on research plots B1 to B10 in the studied area (Šumava National Park) using Ward's method (Bednařík, Matějka 2011) was compared with the first non-metric multidimensional scaling (NMS) ordination axis (bars; compare Fig. 4)

DISCUSSION

Comparison with other methods is problematic. As already mentioned in the introduction, the use of multidimensional statistical methods is not common for dendrometric data, and a comparison with commonly used methods (e.g. ANOVA, *t*-test) may be misleading because the goal of the methods is different. For instance, a statistically significant difference in tree height of two populations may not mean that the individual habitus of the two populations differs, but may be due to different ages. Consequently, the differences in the individual characteristics between the plots surveyed (Table 1) have not been tested. E.g., the most similar plots B4 and B5 (Fig. 3b) differ using the *t*-test at a level less than 2% in all variables without height of crown base, but the shape of trees is similar in both plots (data correlation structure is similar).

The data model in ecology can be viewed in several forms as defined by Cattell (1952). Classical model is described by matrix objects × descriptors × times (Legendre, Legendre 2012). The above presented viewpoint is slightly different, because objects (trees in our case) are clumped into clusters [here represented by (sub)populations]. A better approach can be represented by a model where objects are equal to (sub)populations and descriptors are defined in an appropriate manner. In the presented case, descriptors are represented by individual correlation matrices.

Typical classification of populations is carried out on the basis of genetics. A specific distance measure was proposed for this objective (Nei 1978). The processed data consists of the allele frequencies within a set of loci.

Broad sets of biometric data are used for distinguishing taxa through several populations. The use of the set of quantitative data is evaluated rather by basic statistics (mean, standard deviation etc.) over each population than on the basis of correlation structure of gathered data. Differences between populations would frequently be studied by some multidimensional analyses such as PCA – a few plant taxonomical examples from the Czech republic have been published in the last years (Koutecký et al. 2012; Letz et al. 2012; Budzáková et al. 2014; Lepší et al. 2015), principal coordinates analysis (Olšavská et al. 2015), correspondence analysis, discriminant analysis (Budzáková et al. 2014; Kabátová et al. 2014; Lepší et al. 2015) or hierarchical classification (Csiky et al. 2010). Quantitative leaf morphological data were compared with genetic distances for instance in oaks (Gugerli et al. 2007). Unfortunately, such an approach is more sensitive to the size of individuals than to relationships between measured variables. This approach applied to data like tree height, DBH or crown area would be more dependent on the tree population age than on the shape of trees.

Another method can be developed *ad hoc* for evaluation of the shape of specific organisms, e.g. algae (Marvan, Hindák 1989). These methods are taxa-group dependent and very sensitive to the user definition of variables to be measured.

CONCLUSIONS

The new method is based on the matrices of correlation coefficients between dendrometric characteristics measured for each population (subpopulation) of tree species. Due to the use of correlation coefficients of measured values instead of own variables, the result is not so dependent on the size of the trees, but rather on their "shape" properties, which are determined by the particular environmental conditions and the stand features (primarily by canopy density). The method can be applied to any tree species in any region. This method can be modified for a set of quantitative data describing several populations (subpopulations) of any plant or animal species.

Classification by UPGA and NMS ordination using distances expressed by X^2 variables provide mutually compatible results.

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