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Nonlinear mixed effect height-diameter model for mixed species forests in the central part of the Czech Republic

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ABSTRACT: Various forest models that estimate volume, site index, growth and yield, biomass, and sequestrated carbon amounts are based on the information of the tree heights. The tree heights are obtained either directly from measurements or indirectly estimated using height-diameter models. We developed a nonlinear mixed effect height-diameter model applicable to both conifer and broadleaved tree species through the introduction of dummy variable that accounts for the variations in the height-diameter relationship, caused by the effects of species-specific differences. Data from 255 sample plots located within the multi-layered mixed species forests in the central part of the Czech Republic were used. Based on the fit statistics of twelve bi-parametric models, the Näslund's model, which best fits height-diameter data of various species, was selected for expansion by incorporating height of the tallest tree per sample plot, dummy variable, and sample plot-level random effects. As compared to the ordinary least square model, the mixed effect model described significantly a larger part of the variations in the height-diameter relationship and showed a higher prediction accuracy. Large prediction errors still occurred for the mixed species stands when all measured heights other than the focused species (species used in species group-specific model) per sample plot were used to predict random effects and localize the mixed effect model. But those errors were significantly reduced when all measured heights per sample plot, regardless of species were used to predict random effects. We therefore recommend a mixed effect model with random effects predicted using all measured heights per sample plot, regardless of species, to accurately predict the missing height measurements.

Keywords: height-diameter relationship; multi-layered forest stand; model localization; Näslund's model; prediction error

Tree height and diameter measurements are basic input information to various forest models that serve as important tools in forest management. Various forest models such as volume, site index, growth and yield, biomass, and carbon budget models are based on the measurements of tree heights. For any sample plot inventory design such as national forest inventory sample plots, permanent sample plots, and temporary sample plots, diameters are measured for all trees, but only few selected trees are measured for total heights. Measuring heights of the standing trees is much more difficult, time consuming, and costlier than measuring diameters. In such a situation, a sample plot-specific height-diameter model is necessary for accurate prediction of the missing height measurements of the trees for which diameters are measured.

The height-diameter relationship differs from one stand to another due to differences in site quality, stand age, silvicultural treatments applied, and even within the same stand due to a differing competitive situation among the trees (Vanclay 1994; Zeide, Curtis 2002; Pretzsch 2009). Thus, height-diameter models need to be made stand-specific in order to increase the prediction accuracy. In addition, more realistic description of the forest structure such as growth simulation and estimation of sample plot-level volume is possible with stand specific height-diameter models (Mehtätalo et al. 2015). However, it would be costly and time consuming

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to develop separate stand-specific height-diameter models for a number of stands that are located across the forests. Heights and diameters of at least 20–25 trees per stand need to be measured to develop the accurate stand-specific height-diameter models (van Laar, Akça 2007). This could be possible when there are only a few stands to be covered and the height-diameter models specific to such stands can be developed using either non-parametric approach or parametric approach – a common approach. A single height-diameter model applicable to a large forest area can be developed using one of the two parametric approaches.

The first approach uses site quality measures (site index or dominant height) and stand density measures (basal area, stem number, mean diameter) as additional covariates in the height-diameter models (Sharma, Zhang 2004; Temesgen, von Gadow 2004; Castedo Dorado et al. 2006). These models are commonly known as the generalized ordinary least square height-diameter models. The second approach uses site quality and stand density measure as additional covariates and subject-specific effects as random effects in the height-diameter models (Mehtätalo 2004; Crecente-Campo et al. 2010; Mehtätalo et al. 2015; Sharma, Breidenbach 2015). These models are often termed as the generalized mixed effect models (hereafter termed as mixed effect model) and are more accurate than the generalized ordinary least square height-diameter models (Crecente-Campo et al. 2010; Paulo et al. 2011; Sharma, Breidenbach 2015). This study applies the second approach using stand measures and sample plot-specific random effects. The mixed effect model would therefore have a high prediction accuracy (Robinson, Wykoff 2004; Crecente-Campo et al. 2010; Adamec 2015; Mehtätalo et al. 2015).

This study uses data from the multi-layered mixed species forests. The height-diameter relationship varies more significantly in the multi-layered mixed species stands than in a single species and/or a single-layered stands due to the effects of species-specific differences. These effects can be modelled using a dummy variable approach (Chapagain et al. 2014; Temesgen et al. 2014). When there is a data limitation, developing a species-specific height-diameter model for each of the several species that are present in the same sample plot is not possible. In such a situation, a single height-diameter model applicable to both conifer and broadleaved species can be developed using a dummy variable modelling approach. Stand variable, dummy variable, and sample plot-level random effects have been included in the model. The proposed model can be used for the accurate prediction of missing height measurements on each sample plot.

MATERIAL AND METHODS

Study area. This study was conducted in the forest stands in Prague located in the middle part of the Czech Republic (Fig. 1). The forest stands are located at the altitudes between 178 and 381 m a.s.l., covering 5,100 ha (10% of the Prague area), and out of which 2,900 ha is owned by the capital city of Prague. The mean annual temperature and precipitation of Prague are 8.6°C and 530 mm, and during the growing season they are 8.8°C and 390 mm, respectively. The forest stands are often located on sloping terrains and rocky hillsides, where geological bedrock is much diversified, mostly composed of siltstone, limestone and slate, and soil conditions are fairly heterogeneous. A part of the forest (2,200 ha) falls within the protected area system (Nature Reserve, Natural Monument, Natural Park), which is a unique feature of Prague, both from geological and biological points of view. The forest stands consist of 75% of broadleaved species and the rest is conifer ones. Sessile oak (*Quercus petraea* (Mattuschka) Lieblein) is a dominating broadleaved species while European larch (*Larix decidua* Miller) is a dominating conifer species of the forests.

Research sample plots. Based on the canopy structures, natural regeneration, and stocks of dead wood, 272 circular research plots, hereafter termed as sample plots (area: 400 m^2) were established across the forests (Fig. 1) following the Field-Map technology of the IFER – Monitoring and Mapping Solutions, Ltd. (IFER 2016). Sample plots are located in 250 × 250 m grids across the forests. The position of all trees and regenerations on each sample plot was recorded. Over bark DBH for all trees with DBH ≥ 7 cm were measured with a calliper to the nearest 1 mm. Based on the DBH classes, tree height of at least five sample trees per sample plot for a dominating species and one for each of the other species were measured to the nearest 0.1 m using a laser Vertex hypsometer (Haglöf Sweden, A.B., Långsele, Sweden; Haglöf Sweden, A.B. 2011). The trees with measured heights, hereafter, are termed as height sample trees. Thus, height sample trees represent all DBH classes on each sample plot, covering all diameter classes, but might or might not have included the tallest tree on the sample plot. Other tree- and stand-level variables on each sample plot were also measured following

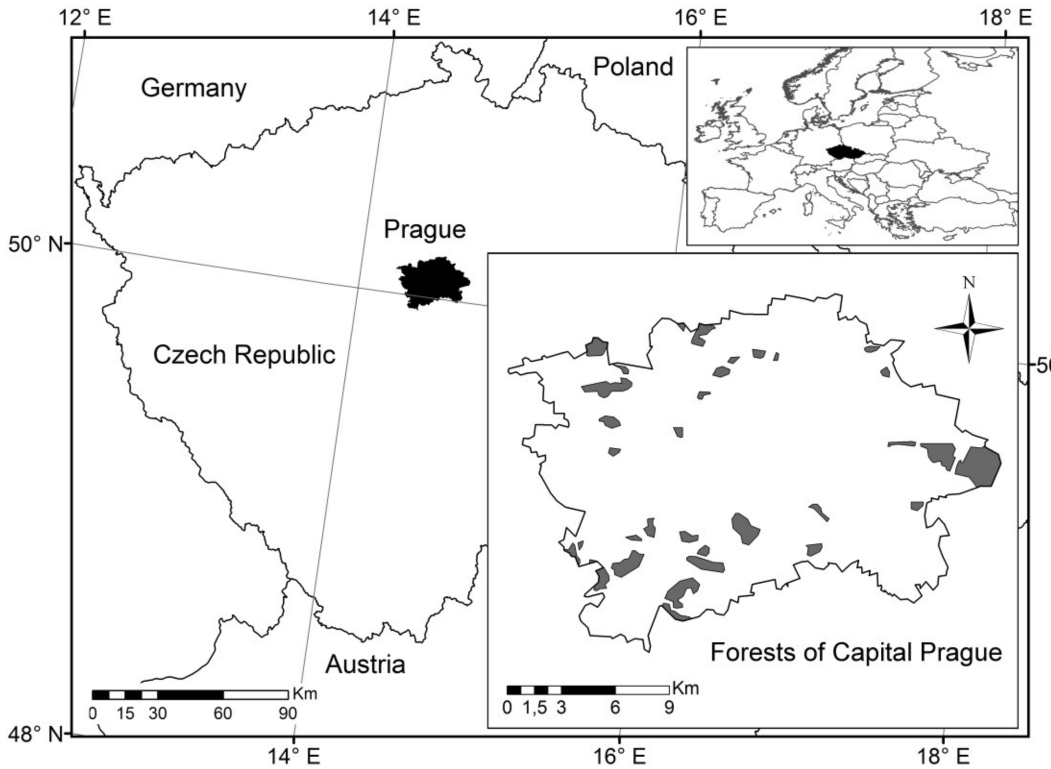

Fig. 1. Various forest stands of Prague (grey colour) where research sample plots are located

the inventory protocols developed by the Forest Management Institute (FMI 2003). Measurements were done between June 2012 and May 2014, but no repeated measurements involved.

Stand variables. Various stand variables that describe site quality and stand density were evaluated in the height-diameter model. However, the variables which describe site quality, such as site index (Monserud 1984) and dominant heights (Sharma et al. 2011), were not measured. Both tree growth and stand dynamics are linked to dominant height as this reflects the site quality (Monserud 1984). Dominant height and dominant diameter are therefore commonly used as additional covariates in the height-diameter models (Castedo Dorado et al. 2005, 2006; Crecente-Campo et al. 2010) and other individual tree models (Fu et al. 2013). However, in the absence of dominant height and dominant diameter, we selected the height and diameter of the tallest tree among the height sample trees per sample plot and used them as a proxy for dominant height (h_{max}) and dominant diameter (D_{max}), respectively (Sharma, Parton 2007; Sharma, Breidenbach 2015). In addition, we also used other stand variables such as stand basal area (BA), number of stems (N), quadratic mean diameter (QMD), and arithmetic mean diameter (AMD) per sample plot that described the stand density effects on the height-diameter relationships. The stand variables which describe the competitive situation among the individual trees within a stand have frequently been included in the height-diameter models (Calama, Montero 2004; Sharma, Zhang 2004; Newton, Amponsah 2007; Adame et al. 2008; Schmidt et al. 2011; Mehtätalo et al. 2015). We also calculated the diameter difference of the thickest and thinnest trees (DBH_{range}) and the height difference of the tallest and shortest trees (h_{range}). All aforementioned stand-level variables were computed using all trees per sample plot regardless of species.

Height-diameter data. We used only height and diameter measurements from the standing, living, and undamaged trees. Except 56% of sample plots, other sample plots consisted of mixed tree species (i.e. at least two tree species existing together). Mixed species sample plots consist of various tree species in general, but in particular, mainly three conifer species such as Norway spruce, Scots pine, and European larch, and four broadleaved species such as oak, maple, ash, and small-leaved linden (Appendix). We excluded height-diameter measurements of only a few trees, which had very small DBH (< 8 cm) but extremely tall heights (> 20 m), assuming that they were due to measurement or recording errors. But, these trees were excluded after stand-level competition measures were computed. Because of applying various data selection criteria, only 94% of sample plots remained for modelling. Summary statistics of the data are presented in Ta-

Table 1. Summary statistics of tree and stand attributes

Variable	Conifer	Broadleaved
Number of sample plots	5 pure + 44 conifer dominated	139 pure + 67 broadleaved dominated
Number of height sample trees	424	1,497
	statistics [mean ± SD (range)]	
Number of height sample trees per sample plot	6.21 ± 1.9 (5–12)	7.2 ± 2.22 (5–15)
Number of trees (*N* per ha)	300 ± 85 (60–500)	258 ± 86 (40–520)
BA ($m^2 \cdot ha^{-1}$)	22.31 ± 8.83 (4.4–50.8)	20.52 ± 13.04 (1–97.3)
QMD (cm)	30.9 ± 6.1 (19.3–48.9)	31.3 ± 9.8 (8.4–72.3)
AMD (cm)	28.7 ± 6.7 (15.8–71.1)	29.4 ± 9.4 (8.3–71.1)
h_{max} (m)	24.7 ± 4.6 (14.8–33.2)	23.3 ± 5.4 (8–38.2)
D_{max} (cm)	38.5 ± 8.2 (17.3–66.7)	40.3 ± 13.1 (7.5–92.2)
DBH_{range} (cm)	28.1 ± 9.2 (8.3–65.1)	29.3 ± 13 (8–69.2)
h_{range} (m)	10.6 ± 5.1 (0.3–25.2)	10 ± 5.7 (0.5–30)
Total height (m)	21.8 ± 5.2 (9.7–33.2)	18.9 ± 6.3 (4.4–38.2)
DBH (cm)	31.7 ± 10.4 (11.1–74.2)	30.3 ± 14.1 (7.1–80.5)

N – number of stems, BA – stand basal area, QMD – quadratic mean diameter per sample plot, AMD – arithmetic mean diameter per sample plot, h_{max} – maximum height per sample plot, D_{max} – maximum diameter per sample plot, DBH_{range} – diameter range per sample plot, h_{range} – height range per sample plot, SD – standard deviation

ble 1. The number of height sample trees per sample plot varied from 5 to 15. The pairs of height and diameter measurements are shown in Fig. 2. Most of the observations for each species group seem to have occupied the same space in this graph.

Base models. Generally, relationship between height and diameter of a tree is nonlinear and height curve increases more rapidly in earlier stages than in later stages (Lappi 1997; Pretzsch 2009; Schmidt et al. 2011). Our data also show a significant nonlinear pattern (Fig. 2). We therefore selected twelve bi-parametric nonlinear models (Table 2) to fit data. We examined the fit statistics of each model and selected the one which showed the smallest sum of squared errors. Choosing only bi-parametric models was that the convergence of each model for each individual sample plot must be ensured. Näslund's model (Näslund 1936) showed the smallest sum of squared errors of fitting data, and therefore was selected for further analyses. Other studies (Kangas, Maltamo 2002; Schmidt et al. 2011; Mehtätalo et al. 2015; Sharma, Breidenbach 2015) also used this model for further expansion because of its adequate flexibility. This model, hereafter termed as a base model, is given by Eq. 1:

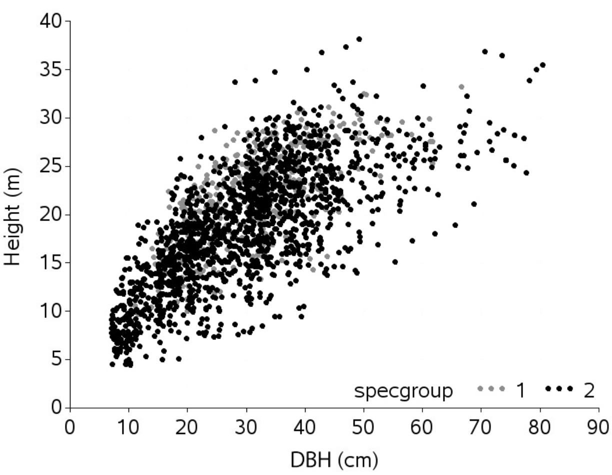

Fig. 2. Total height against diameter; specgroup 1 – conifer tree species, specgroup 2 – broadleaved tree species

$$h_{ij} = 1.3 + \left(\frac{DBH_{ij}}{b_1 + b_2 DBH_{ij}}\right)^3 + \varepsilon_{ij} \quad (1)$$

where:

h_{ij} – height measurement for tree *j* (*j* = 1, ..., *m*) on sample plot *i* (*i* = 1, ..., *n*),

1.3 – added to avoid the prediction of zero height when DBH approaches zero,

DBH_{ij} – diameter measurement for tree *j* (*j* = 1, ..., *m*) on sample plot *i* (*i* = 1, ..., *n*),

b_1, b_2 – parameters to be estimated,

ε_{ij} – residual error.

Selection of stand variables. Base model (Eq. 1) described only a small part of the variations in the height-diameter relationship when fitted separately to a species group-specific (conifer species group and broadleaved species group) data. Thus, this model would be too poor to be applied for height predictions. We therefore included stand variables in order to describe a larger part of the variations in the height-diameter relationship. The stand variables can be included in the model using one of the

Table 2. Base models used to fit height-diameter data

Designation	Model	Reference
M1	$h_{ij} = 1.3 + b_1 \mathrm{DBH}_{ij}^{b_2} + \varepsilon_{ij}$	HUXLEY and TEISSIER (1936)
M2	$h_{ij} = 1.3 + b_1[1 - \exp(-b_2 \mathrm{DBH}_{ij})]^3 + \varepsilon_{ij}$	BERTALANFFY (1957)
M3	$h_{ij} = 1.3 + \left[\frac{\mathrm{DBH}_i}{(b_1 + b_2 \mathrm{DBH}_{ij})}\right]^3 + \varepsilon_{ij}$	NÄSLUND (1936)
M4	$h_{ij} = 1.3 + \frac{b_1 \mathrm{DBH}_{ij}}{(b_2 + \mathrm{DBH}_{ij})} + \varepsilon_{ij}$	HUANG et al. (2000)
M5	$h_{ij} = 1.3 + b_1[1 - \exp(-b_2 \mathrm{DBH}_{ij})] + \varepsilon_{ij}$	MEYER (1940)
M6	$h_{ij} = 1.3 + b_1 \left[\frac{\mathrm{DBH}_{ij}}{1 + \mathrm{DBH}_{ij}}\right]^{b_2} + \varepsilon_{ij}$	HUANG et al. (2000)
M7	$h_{ij} = 1.3 + \exp\left[b_1 + \frac{b_2}{(\mathrm{DBH}_{ij} + 1)}\right] + \varepsilon_{ij}$	WYKOFF et al. (1982)
M8	$h_{ij} = 1.3 + \frac{b_1 \mathrm{DBH}_{ij}}{(\mathrm{DBH}_{ij} + 1) + b_2 \mathrm{DBH}_{ij}} + \varepsilon_{ij}$	BATES and WATTS (1980)
M9	$h_{ij} = 1.3 + b_1 \mathrm{DBH}_{ij} \exp(-b_2 \mathrm{DBH}_{ij}) + \varepsilon_{ij}$	HUANG et al. (2000)
M10	$h_{ij} = 1.3 + \exp\left(b_1 + \frac{b_2}{\mathrm{DBH}_{ij}}\right) + \varepsilon_{ij}$	STAUDHAMMER and LEMAY (2000)
M11	$h_{ij} = 1.3 + b_1 \exp\left(\frac{b_2}{\mathrm{DBH}_{ij}}\right) + \varepsilon_{ij}$	BUFORD (1986)
M12	$h_{ij} = 1.3 + b_1 \left(1 + \frac{1}{\mathrm{DBH}_{ij}}\right)^{-b_2} + \varepsilon_{ij}$	CURTIS (1967)

h_{ij} – height measurement for tree j (j = 1, ..., m) on sample plot i (i = 1, ..., n), 1.3 – added to avoid the prediction of zero height when DBH approaches zero, b_1, b_2 – parameters to be estimated, DBH_{ij} – diameter measurement for tree j (j = 1, ..., m) on sample plot i (i = 1, ..., n), ε_{ij} – residual error

two approaches (HUANG, TITUS 1994; STAUDHAMMER, LEMAY 2000). The first is the parameter prediction approach (CLUTTER et al. 1983), and is also known as a two-stage approach (FERGUSON, LEECH 1978), which indirectly adds stand variables into the model, and the second approach directly adds stand variables into the models (SHARMA, ZHANG 2004; SHARMA, PARTON 2007; SCHMIDT et al. 2011). Because of its biological relevance leading to easier interpretation (STAUDHAMMER, LEMAY 2000; SHARMA et al. 2016), we applied two-stage approach. This is commonly used to select appropriate predictor variables (MEHTÄTALO 2004; CASTEDO DORADO et al. 2005; ADAME et al. 2008; SHARMA, BREIDENBACH 2015). In the first stage, we fitted the base model to the data for each sample plot separately and sample plot-specific estimates of the parameters (b_1, b_2) of a base model were then plotted against each stand variable (N, BA, h_{max}, D_{max}, QMD, AMD, DBH_{range}, h_{range}). Matrix plots of the relationship between each parameter of a base model against each of these stand variables and their transformations (square, logarithm, and root) and interactions were examined. In the second stage, the stand variables which showed a strong relationship with b_1 or b_2 were used as covariates to expand the base model (Eq. 1). As compared to other variables, only h_{max} showed a strong relationship with b_2 of a base model. We then expanded the base model through redefining its parameter b_2 as a function of h_{max}. Other stand variables were also subsequently added to this expanded model, however, no significant improvement was observed.

Many of the stand variables tested in this study, such as N, BA, QMD, AMD, DBH_{range} and h_{range}, are influenced by thinning. The prediction accuracy for the same stand differs significantly before and after thinning even if the same height-diameter model is applied. The chosen stand variables to develop a height-diameter model should therefore be independent of thinning. The h_{max} is more independent of thinning (except thinning from above) than other stand variables.

We coded two species groups (conifer and broadleaved species) as 0 and 1 and developed a height-diameter model using the dummy variable modelling approach. This approach accounts for the variations in the height-diameter relationship, caused by the effect of species group-specific differences (Chapagain et al. 2014; Temesgen et al. 2014). The effect of species group-specific differences was best described by b_1 of a base model when it was expressed as a linear function of a species group. The expanded version of Eq. 1 with dummy and stand variables is given by Eq. 2:

$$h_{ij} = 1.3 + \left(\frac{\mathrm{DBH}_{ij}}{(b_1+b_2)\mathrm{DBH}_{ij}}\right)^3 + \varepsilon_{ij} \qquad (2)$$

where:

h_{ij} – height measurement for tree j (j = 1, ..., m) on sample plot i (i = 1, ..., n),

1.3 – added to avoid the prediction of zero height when DBH approaches zero,

DBH_{ij} – diameter measurement for tree j (j = 1, ..., m) on sample plot i (i = 1, ..., n),

b_1, b_2 – parameters to be estimated ($b_1 = a_1 + a_2$ species, if species is broadleaved, species = 1, otherwise 0; $b_2 = a_3 h_{max}^{a_4}$),

a_1–a_4 – parameters to be estimated,

h_{max} – the tallest height among the measured heights per sample plot, regardless of species.

Nonlinear mixed effect model. We applied a mixed effect modelling approach by incorporating sample plot-level random effects into the model. A one-level nonlinear mixed effect model (Pinheiro, Bates 2000) is generally represented by Eq. 3:

$$y_i = f(\theta_i, x_i) + \varepsilon_i \qquad (3)$$

where:

y_i – response vector for height measurements,

θ_i – parameter vector of a nonlinear model ($\theta_i = A_i b + B_i u_i$),

b – vector of fixed parameters with design matrix A_i,

B_i – random-effects design matrix for sample plot i,

u_i – vector of sample plot-level random effects (u_{i1}, u_{i2}) for sample plot i, it is assumed to have multivariate normal distribution with zero mean and variance-covariance matrix D ($u_i \sim N(0, D)$),

x_i – predictor vector for DBH measurements on sample plot i,

ε_i – residual vector ($\varepsilon_i \sim N(0, R_i)$),

$\sim N$ – normally distributed with zero mean and within-sample-plot variance-covariance matrix R_i.

The within-sample-plot variance-covariance matrix (R_i) is given by Eq. 4:

$$R_i = \sigma^2 G_i^{\frac{1}{2}} \Gamma_i G_i^{\frac{1}{2}} \qquad (4)$$

where:

σ^2 – residual variance common to all sample plots, i.e. σ^2 is a scaling factor for error dispersion (Gregoire et al. 1995) given by the value of the residual variance of the estimated model,

G_i – diagonal matrix describing the variance of within-sample-plot residual heteroscedasticity,

Γ_i – matrix accounting for within-sample-plot autocorrelations of the residuals, however, Γ_i was reduced to an identity matrix because of the absence of within-sample-plot autocorrelations.

Among three alternative variables (DBH, observed height, estimated height) tested to stabilize the variance, within-sample-plot residual heteroscedasticity was taken into account by modelling variance as a function of the estimated height, as Eq. 5:

$$\mathrm{var}(\varepsilon_i) = \sigma^2 \hat{h}_{ij}^2 \qquad (5)$$

where:

ε_i – residual vector ($\varepsilon_i \sim N(0, R_i)$),

σ^2 – residual variance common to all sample plots, i.e. σ^2 is a scaling factor for error dispersion (Gregoire et al. 1995) given by the value of the residual variance of the estimated model,

h_{ij} – height measurement for tree j (j = 1, ..., m) on sample plot i (i = 1, ..., n).

The mixed effect model after inclusion of sample plot-level random effects is given by Eq. 6:

$$h_{ij} = 1.3 + \left(\frac{\mathrm{DBH}_{ij}}{b_1+u_{i1}+(b_2+u_{i2})\mathrm{DBH}_{ij}}\right)^3 + \varepsilon_{ij} \qquad (6)$$

where:

h_{ij} – height measurement for tree j (j = 1, ..., m) on sample plot i (i = 1, ..., n),

1.3 – added to avoid the prediction of zero height when DBH approaches zero,

b_1, b_2 – parameters to be estimated ($b_1 = a_1 + a_2$ species, if species is broadleaved, species = 1, otherwise 0; $b_2 = a_3 h_{max}^{a_4}$),

u_i – vector of sample plot-level random effects (u_{i1}, u_{i2}) for sample plot i, it is assumed to have multivariate normal distribution with zero mean and variance-covariance matrix D ($u_i \sim N(0, D)$),

DBH_{ij} – diameter measurement for tree j (j = 1, ..., m) on sample plot i (i = 1, ..., n),

ε_i – residual vector ($\varepsilon_i \sim N(0, R_i)$),

$\sim N$ – normally distributed with zero mean and within-sample-plot variance-covariance matrix R_i.

Hereafter, a model estimated with random effects (Eq. 6) and that without random effects (Eq. 2) are termed as mixed effect model and ordinary least square (OLS) model, respectively.

Model estimation and evaluation. The mixed effect model was estimated with restricted maximum likelihood in SAS macro NLINMIX (SAS Institute Inc. 2008) using an expansion-around-zero method (Littell et al. 2006). Base models (Table 2) were estimated using PROC MODEL (SAS Institute Inc. 2008) with the ordinary least square method. The estimated models were evaluated using root mean square error (RMSE), adjusted coefficient of determination (R^2_{adj}), Akaike information criterion (AIC) (Akaike 1974) and Schwarz's Bayesian information criterion (BIC). Graphs of the residuals and simulated curves overlaid on the observed data were also examined for each species group. We also examined prediction errors of the subject-specific (or localized) model and height-diameter curves overlaid on the observed data for each sample plot.

Subject-specific predictions. In addition to predictor variables, a priori information of a response variable (Eq. 6) is needed for subject-specific predictions (Pinheiro, Bates 2000; Calama, Montero 2004). The measured heights of any number of trees per sample plot can be used to predict random effects and adjusted to the fixed part of the mixed effect model, and this is also known as localization or calibration of the mixed effect model. We applied the empirical best linear unbiased prediction (EBLUP) theory (Eq. 7) (Vonesh, Chinchilli 1997; Pinheiro, Bates 2000) to estimate sample plot-level random effects in Eq. 6 using PROC IML of SAS (SAS Institute Inc. 2008):

$$u_i = DZ_i^T(Z_iDZ_i^T + R_i)^{-1}\,\varepsilon_i \qquad (7)$$

where:

u_i – vector of sample plot-level random effects (u_{i1}, u_{i2}) for sample plot i, it is assumed to have multivariate normal distribution with zero mean and variance-covariance matrix D ($u_i \sim N(0, D)$),

Z_i – design matrix for the random effects specific to additional observations, the elements of matrix Z_i are partial derivatives of the nonlinear model (Eq. 6) with respect to its fixed parameters vector b (Calama, Montero 2004; Crecente-Campo et al. 2010; Sharma, Breidenbach 2015),

Z^T – transpose matrix,

R_i – within-sample-plot variance-covariance matrix,

ε_i – residual vector ($\varepsilon_i \sim N(0, R_i)$),

$\sim N$ – normally distributed with zero mean and within-sample-plot variance-covariance matrix R_i.

Our main objective was to correctly impute the missing heights on the same sample plots that were used in this study. Therefore, instead of using splitting data or getting new data from different forest stands for the model validation, we used fitting data to predict random effects using the EBLUP theory (Eq. 7) in order to localize the mixed effect model and we examined the prediction errors for each subject (or sample plot). For this purpose, we used various alternative methods that involved the selection of differing numbers of trees systematically or randomly with respect to heights, and prediction of random effects using the measured heights of the selected trees. Those alternatives are: systematically selected one shortest, one medium and one tallest height sample tree (alternative 1 to 3), and randomly selected one to seven height sample trees (alternative 4 to 10). This evaluation was possible only for 167 sample plots where more than six height sample trees per sample plot were available. We also examined the prediction errors of the localized model using measured heights of the trees other than the focused species (species used in a group-specific model).

RESULTS

The base model (Eq. 1) described only a small part of the variations in the height-diameter relationship when fitted separately to species group-specific data ($R^2_{adj} < 0.55$). To improve the prediction accuracy, stand-level variable (h_{max}), dummy variable (species) and sample plot-specific random effects were included in the base model. All parameter estimates and variance components of the expanded models are highly significant ($P < 0.0001$) (Table 3). Both OLS model and mixed effect model described the variations in the height-diameter relationship adequately well, but as expected, the former model fitted more poorly as compared to the latter model. The stand variable contributed significantly highly to the description of the variations in the height-diameter relationship is the tallest height per sample plot (h_{max}). The inclusion of within-sample-plot residual heteroscedasticity through variance modelling was able to reduce AIC by 2–3%. A reduction of the unexplained variance (i.e. mean squared residual, σ^2) in the mixed effect model relative to the OLS model is 43%. A higher estimated value of the random effect parameter u_{i1} suggested that parameter b_1 was more strongly correlated with the sample plot-level variations than parameter b_2.

Table 3. Parameter estimates, variance-covariance components, and fit statistics of ordinary least square (OLS) model (Eq. 2) and its mixed effect version (Eq. 6)

Component	OLS model	Mixed model
Fixed		
a_1	1.0230	0.9703
a_2	0.3283	0.3629
a_3	0.8960	0.9860
a_4	–0.3126	–0.3424
Variance and covariance		
σ^2_{ui1}		0.3065
$\sigma_{ui1\,ui2}$		–0.00571
$\sigma^2_{ui1\,ui2}$		0.000141
σ^2	0.02322	0.01163
Fit statistic		
RMSE	2.5358	1.8941
R^2_{adj}	0.8289	0.9045
AIC	9,428	8,691
BIC	9,456	8,719
AIC reduction after inclusion of heteroscedasticity (%)	3	2

a_1–a_4 – parameters to be estimated, σ^2 – residual variance common to all sample plots, i.e. σ^2 is a scaling factor for error dispersion (Gregoire et al. 1995) given by the value of residual variance of the estimated model, u_i – vector of sample plot-level random effects (u_{i1}, u_{i2}) for sample plot *i*, it is assumed to have multivariate normal distribution with zero mean and variance-covariance matrix *D* ($u_i \sim N(0, D)$), RMSE – root mean square error, R^2_{adj} – adjusted coefficient of determination, AIC – Akaike information criterion, BIC – Schwarz's Bayesian information criterion

We evaluated prediction errors of the mixed effect model using predicted random effects with the EBLUP method from differing number of height sample trees (Table 4). The prediction accuracy of the mixed effect model increased with increasing number of height sample trees used to predict random effects. The shortest tree also resulted in higher accuracy than the medium or the tallest tree. The smallest prediction error was produced when random effects were predicted using all measured heights per sample plot, no matter whether it was a pure species or mixed species sample plot. The rate of reduction of RMSE relative to the fixed part of the mixed effect model decreased with increasing numbers of trees, but the rate seemed too small to be insignificant after four trees. As compared to the fixed part of the mixed effect model, the OLS model showed slightly higher prediction accuracy.

We also examined the prediction accuracy of the mixed effect model using data from only mixed species sample plots. Except the shortest height classes, the mixed effect model was able to show smaller prediction errors for all other height classes when all measured heights of the focused species were used to predict random effects (Figs 3a, c). When the measured heights of different species other than the focused species were used to predict random effects, the prediction errors appeared to be relatively larger for both smaller and larger height classes (Figs 3b, d). However, when all measured heights per sample plot, regardless of the species, were used to predict random effects, those larger errors substantially decreased (Figs 4a, b). Except for height classes < 20 m and > 30 m for conifer species, no large errors appeared for pure broadleaved species (Figs 4c, d). As indicated by box length in Figs 3 and 4, most prediction errors are confined to a range of ± 2 m. However, significantly large prediction errors still remained to be accounted for some sample plots, even when all measured heights were used to localize the mixed effect model (Fig. 4).

We also examined height prediction errors and the height-diameter curves produced by the localized mixed effect model for each sample plot. For mixed species sample plots, height-diameter curves are clearly differentiated into two groups: one for conifer species group and other for broadleaved species group, and each curve passes through the middle of the observed data points of the corresponding species group (Fig. 5b). Except for few sample plots, height-diameter curves produced by the localized model showed complete coverage to the observed data for each sample plot. Height-diameter curves overlaid on the observed data for all 225 sample plots are shown in Fig. 5a.

DISCUSSION

Data used in this study vary widely (Table 1, Fig. 2), and represent all possible growth conditions and silvicultural treatments applied in the forests. The height-diameter relationship varies with the development stage of an individual stand due to the effects of stand density (Curtis 1967; Zeide, Curtis 2002) and site quality (Sharma, Zhang 2004). Like many others (Sharma, Parton 2007; Adame et al. 2008; Mehtätalo et al. 2015; Sharma, Breidenbach 2015), we also included stand variables in the height-diameter model to describe a larger part of the variations caused by the effects of site quality and stand density on the height-diameter relationship. Among several stand variables tested (Table 1), we chose only the most significant one using the two-stage variable selection method, which is also

Table 4. Various methods (varying from systematic to random) of selecting height sample trees to predict random effects when localizing the mixed effect model (167 permanent research plots with at least seven height sample trees)

	RMSE	R^2_{adj}	RMSE reduction relative to fixed part of mixed effect model (%)
OLS model	2.5831	0.8173	0.8
Fixed part of mixed effect model	2.6048	0.8142	0
Mixed effect model			
One shortest height sample tree	2.4852	0.8309	4.6
One medium height sample tree	2.5325	0.8244	2.8
One tallest height sample tree	2.6002	0.8125	0.2
One height sample tree	2.5637	0.8201	1.6
Two height sample trees	2.2951	0.8557	11.9
Three height sample trees	2.2004	0.8674	15.5
Four height sample trees	2.1244	0.8764	18.4
Five height sample trees	2.0661	0.8831	20.7
Six height sample trees	2.0209	0.8882	22.4
Seven height sample trees	2.0079	0.8896	22.9

OLS model – ordinary least square model, RMSE – root mean square error, R^2_{adj} – adjusted coefficient of determination

considered as the most relevant one (Staudhammer, LeMay 2000; Castedo Dorado et al. 2005; Adame et al. 2008; Sharma et al. 2016). The selected stand variable (h_{max}), which showed the largest contribution to the model, is also independent of thinning, except the case involving "thinning from above". This is the reason why we used h_{max} as a covariate in our height-diameter model, and

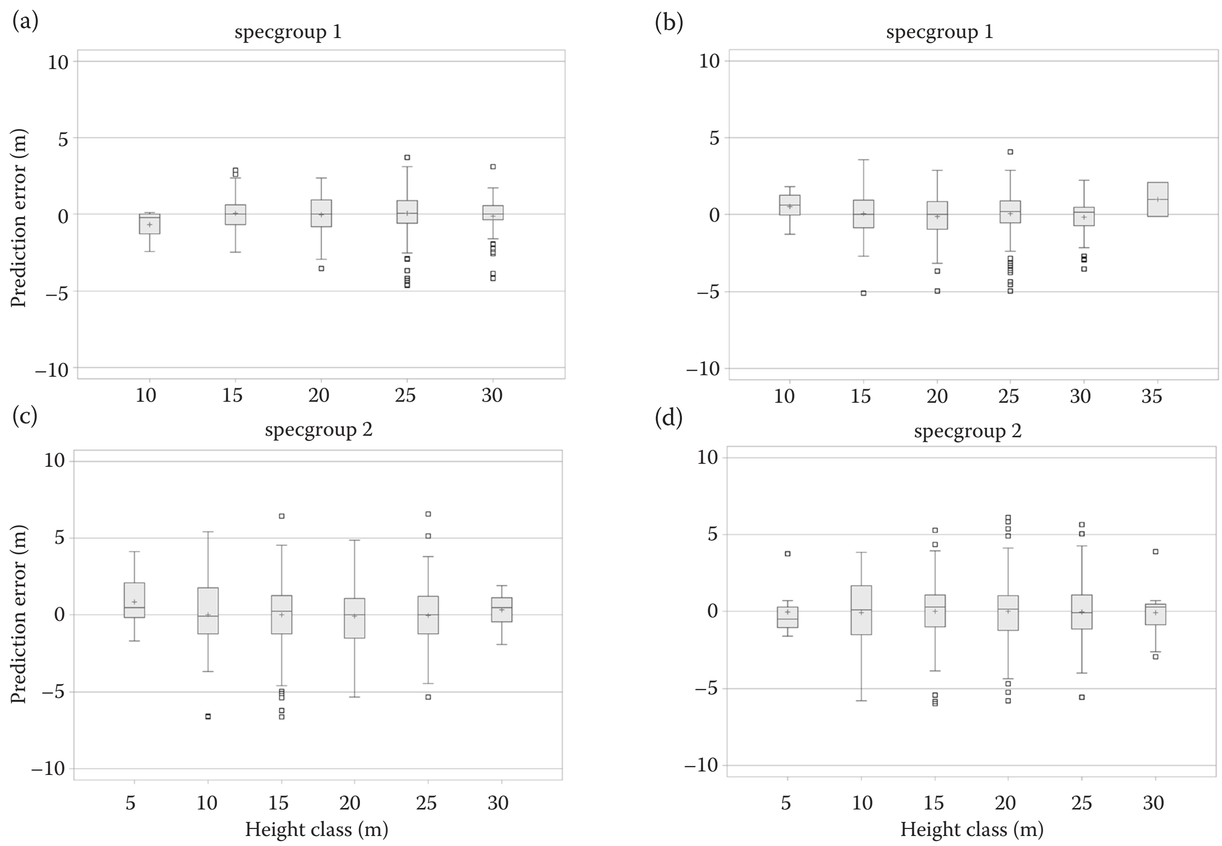

Fig. 3. Prediction errors of the mixed effect model for mixed species sample plots (111 sample plots) using all measured heights of the focused species (a, c), and other than the focused species (b, d). Here, predicted heights were used for height classes

specgroup 1 – conifer tree species, specgroup 2 – broadleaved tree species, length of the box – interquartile range (IQR), length of the whisker – class minimum and maximum values in IQR, small boxes – observations lying beyond 1.5 times IQR

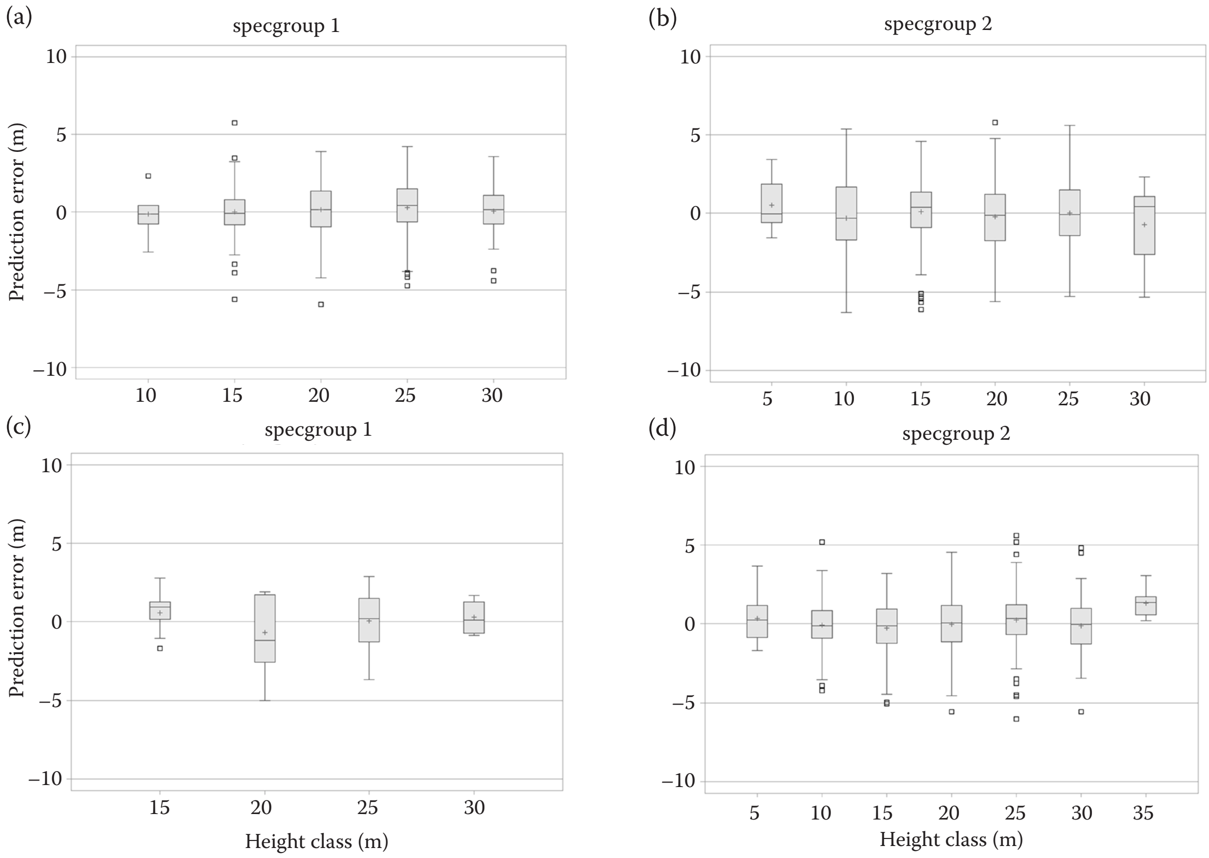

Fig. 4. Prediction errors of the mixed effect model for mixed species sample plots (111 sample plots) (a, b), and for pure species sample plots (144 sample plots) (c, d) using all measured heights per sample plot, regardless of the species. Here, predicted heights were used as height classes

specgroup 1 – conifer tree species, specgroup 2 – broadleaved tree species, length of the box – interquartile range (IQR), length of the whisker – class minimum and maximum values in IQR, small boxes – observations lying beyond 1.5 times IQR

has also been frequently used in other height-diameter models (Sharma, Parton 2007; Sharma, Breidenbach 2015). However, from the model application perspective, using diameter-based stand measures (e.g. mean DBH) may be much cheaper and more practical (Mehtätalo 2004; Mehtätalo et al. 2015) as less effort is required to measure diameter than height. As for the inventory protocols designed by the Forest Management Institute (FMI 2003), the future inventory design is assumed to have the selection of height sample trees based on the diameter classes on each sample plot, covering all diameter classes, for height measurement and the tree with h_{max} among the measured heights can be easily identified. However, this h_{max} tree may or may not be the tallest one (extreme size), because there may be other trees which may include the tallest one, but may not be selected for height measurement. It is thus assumed that model users will not have a problem of getting this covariate information from the future inventories and can accurately predict the missing height measurements of the remaining trees on each sample plot. Our model is not also intended to be used for new sample plots or other stands rather than those used in this study.

Our data originated from the multi-layered mixed species stands and finding the best base model that could largely describe the variations in the height-diameter relationship for each sample plot was quite challenging. Therefore, we performed the evaluation of several base models (Table 2) to find the most appropriate one to our data. Selected model (Näslund's model) is based on the growth theory, i.e. faster increase of height in the earlier stage and slower increase in the later stage (Lappi 1997; Pretzsch 2009; Schmidt et al. 2011). This model is suitable not only for single canopy-layered stands, but also for multi-layered stands when stand-level variables and subject-specific random effects (i.e. sample plot-level random effects) are included in the model (Kangas, Maltamo 2002; Schmidt et al. 2011; Mehtätalo et al. 2015; Sharma, Breidenbach 2015). Because of its pronounced flexibility, Näslund's model also best fits height-diameter data from a number of tree species (Mehtätalo et al. 2015). Our height-diameter model has a pronounced sensitivity, and therefore making a small change in h_{max} results in an apparent change in the height-diameter relationship. Our model also behaves significantly differently for two species groups (Fig. 5). This is due to large effects of species group-specific differences that were successfully modelled (Table 3). A large estimated value of variance u_{i1} suggests that parameter b_1 of the mixed effect model highly varies across the sample plots. This justifies applying the sample plot-level mixed effect modelling approach.

The mixed effect height-diameter model can accurately predict the missing height measurements, but large errors still remain to be accounted for (Table 4, Figs 3 and 4). This is due to the presence of extreme outlier observations that originated from multi-layered stands. Thus, the prediction accuracy of the mixed effect model for each individual tree depends highly on the vertical heterogeneity of a stand and numbers of the measured height sample trees to be used to predict random effects. Measured heights of any number of trees could be used to predict random effects and localize the mixed effect model, but accuracy largely depends on the representativeness of heights of the chosen trees. Generally, higher the number of height sample trees chosen for the prediction of random effects, higher would be the prediction accuracy of the mixed effect model (Table 4). For a stand with homogeneous canopy, measured heights of only a few trees, even a single tree, work adequately well (Trincado et al. 2007). The prediction errors could be significantly reduced for a multi-layered stand, no matter whether the stand is pure or mixed species, when all measured heights were used to predict random effects (Fig. 4). Measured heights of a single tree such as shortest or medium or tallest tree may be used to predict random effects. However, substantial bias may arise when the shortest or the tallest tree is used to predict random effects, because the first alternative most likely leads to underprediction for the tree population and the second leads to overprediction. Our evaluation of the effects of these selection alternatives on the height predictions also indicated such a consequence because of heterogeneities in the vertical structure of the stands. In contrast to selecting only the tallest or the shortest height sample tree systematically, random selection of trees for the prediction of random effects may not cause substantial bias, because those selected trees also correspondingly represent some random diameter classes, mimicking the selection based on diameters. However, it would be more appropriate when heights were randomly selected based on diameter classes rather than height classes, to predict the random effects.

A medium-sized tree may be used for higher prediction accuracy than using the shortest or tallest trees (Crecente-Campo et al. 2010; Sharma, Breidenbach 2015). The rate of reduction of

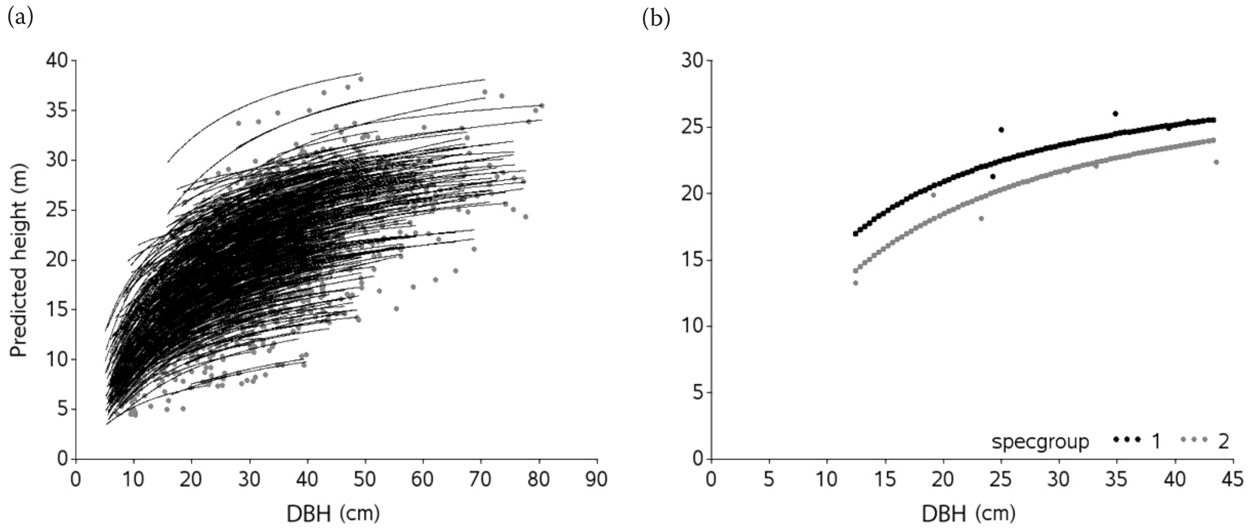

Fig. 5. Trajectories of predicted heights by the mixed effect model for all 255 sample plots (a), with the random effects predicted using all measured heights per sample plot, regardless of the species. Individual dots are observed data. Each mixed species sample plot has two distinct height-diameter curves which represent conifer and broadleaved species, respectively (b)

specgroup 1 – conifer tree species, specgroup 2 – broadleaved tree species prediction errors would be too small to be insignificant when more than four height sample trees were chosen to predict random effects (Table 4). This suggests that four height sample trees may be the optimum number for localizing the mixed effect model. This is also consistent with the results of many other studies (Calama, Montero 2004; Castedo Dorado et al. 2006; Fu et al. 2013). This may also justify balancing the required inventory cost and desired prediction accuracy. The highest accuracy is obtained only when all measured heights per sample plot, regardless of the species, are used to predict random effects.

When measured heights of different species other than the focused species were used to predict random effects, the prediction errors would still be larger (Figs 3b, d). It is due to a significant difference between the heights of the focused species and other species on the same sample plot and chosen height sample trees to predict random effects may not be sufficiently representative to the rest of the trees. The prediction errors for the mixed species stand were significantly reduced when all measured heights regardless of the species are used to predict random effects (Figs 4a, b). Also, in order to reduce prediction errors for pure species stand, which may be uneven-aged or/ and multi-layered, all measured heights per sample plot must be used to predict random effects (Sharma, Breidenbach 2015). For a condition, when model users are not able to predict random effects using EBLUP method due to its complexity, the application of the OLS model rather than only the fixed part of the mixed effect model is suggested (Meng et al. 2009; de-Miguel et al. 2012; Sharma, Breidenbach 2015). However, in recent years, there has been a good access to the advanced computational facilities, which makes the application of the mixed effect model much easier. The height-diameter measurements are made on the same subject (same sample plot in our case) and therefore correlated with each other, and an assumption of independent errors is largely violated on the OLS fitting. This would result in biased variance of the parameter estimates and thus invalidate the hypothesis tests (Vonesh, Chinchilli 1997; Pinheiro, Bates 2000). An appropriate solution to this problem is that one has to use a mixed effect modelling approach to fit data. The predictions are also unbiased when the parameter estimates of the mixed effect model are unbiased and assumptions on the independence and identical distribution of random effects are met. However, the OLS fit often necessarily seems the better, but this is an artifact related to the use of RMSE as a comparison criterion, or to inappropriate formulation of the mixed effect model. Therefore, the mixed effect model needs to be a preferred choice in order to minimize the prediction bias.

There may be a number of methods for calibrating height-diameter models. One of the alternative methods for modelling and calibrating the height-diameter relationship for several tree species may have a model system where each tree species has its own model, and localization or calibration of the mixed effect model is done through the cross-calibration (de Souza Vismara et al. 2016). However, we had very few observations for some species (Appendix) for which developing species-specific height-diameter model was not possible.

The subject-specific prediction bias may arise when a nonlinear mixed effect model is localized using the EBLUP method. There may be four different linearization methods, namely linearization at zero, linearization at the conditional mode, Laplace approximation, and best unbiased prediction with the Metropolis algorithm (Sirkiä et al. 2015). These methods are shown to be evidently different in the theoretical sense and therefore, they yield slightly different results under certain conditions. Such differences may be practically meaningful in a given forest modelling setting and depends on the details of that particular situation and an intended use of the model prediction. In case of a nonlinear mixed effect model, it may be difficult to say how much, or even to which direction, different predictions would differ without actually finding them. In such a situation, for routine updates of forest stand and tree attributes, one may apply the EBLUP method to localize the nonlinear mixed effect model. The literatures also show that most of the forest modellers have used the EBLUP method while localizing the nonlinear mixed effect models, which is practically easier and more suitable than other linearizing methods.

To conclude, the mixed effect model predicted heights more accurately, not only for pure species stands, but also for mixed species stands, when all measured heights per sample plot, regardless of species, were used to predict random effects. The application of our models needs to be restricted to the same sample plots which were used in this study. The missing heights of a large number of trees (7–71% of trees per sample plot) can be imputed more accurately with the mixed effect model rather than applying OLS model or fixed part of mixed effect model (mean response).

Appendix: List of tree species recoded on the sample plots

English name	Scientific name	Number of trees
Conifer species		
European larch	*Larix decidua* Miller	144
Scots pine	*Pinus sylvestris* Linnaeus	120
Australian pine, European black pine	*Pinus nigra* J.F. Arnold	53
Norway spruce	*Picea abies* (Linnaeus) H. Karsten	97
Douglas fir	*Pseudotsuga menziesii* (de Mirbel) Franco	5
Eastern white pine	*Pinus strobus* Linnaeus	5
Broadleaved species		
Sessile oak, durmast oak	*Quercus petraea* (Mattuschka) Lieblein	527
Red oak, northern red oak	*Quercus rubra* Linnaeus	162
European beech	*Fagus sylvatica* Linnaeus	119
European hornbeam	*Carpinus betulus* Linnaeus	117
Norway maple	*Acer platanoides* Linnaeus	105
Sycamore maple	*Acer pseudoplatanus* Linnaeus	75
Field maple, hedge maple	*Acer campestre* Linnaeus	66
European ash, common ash	*Fraxinus excelsior* Linnaeus	62
Elm	*Ulmus* sps.	54
Robinia, locust, black locust	*Robinia pseudoacacia* Linnaeus	47
European birch, common birch	*Betula pendula* Roth	45
European mountain ash, rowan	*Sorbus aucuparia* Linnaeus	24
Mazzard cherry, wild cherry	*Prunus avium* Linnaeus	23
Pedunculate oak, English oak	*Quercus robur* Linnaeus	16
Black cherry	*Prunus serotina* Ehrhart	15
Wild pear	*Pyrus pyraster* (Linnaeus) Burgsdorff	9
Small-leaved linden	*Tilia cordata* Miller	8
Silver lime	*Tilia tomentosa* Moench	7
Black alder, common alder	*Alnus glutinosa* (Linnaeus) Gaertner	6
Aspen	*Populus tremula* Linnaeus	5
Goat willow	*Salix caprea* Linnaeus	3
Horse chestnut	*Aesculus hippocastanum* Linnaeus	2

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