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Comparison of different non-linear models for prediction of the relationship between diameter and height of velvet maple trees in natural forests (Case study: Asalem Forests, Iran)

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ABSTRACT: Velvet maple (*Acer velutinum*) is one of the woody species in the Hyrcanian forests. In this study, the relationship between height and diameter of velvet maple was surveyed. A complete list of the selected height-diameter models was used and nineteen candidate models were considered. Various criteria were chosen and applied to evaluate the predictive performance of the models. These criteria include Akaike information criterion (AIC), Bayesian information criterion (BIC), root mean square error (RMSE), mean error (ME), and adjusted coefficient of determination (R^2_{adj}). Fitting of nineteen height-diameter models using nonlinear least square regression showed that all of the parameters in models were significant ($P < 0.01$). The results of goodness of fit for the calibration and k-fold validation and the performance criteria (RMSE, ME, AIC, R^2_{adj} and BIC) showed that R^2_{adj} ranged from 0.743 (model 8) to 0.8592 (model 11) and RMSE from 2.6983 (model 11) to 10.1897 (model 9). The range of ME among the models is from –7.0787 (model 9) up to 0.063m (model 7). By considering the AIC for each model it is evident that model (11) and model (9) have the lowest and highest values, respectively. Plotting the residuals showed that for all these models the residuals were randomly distributed and the models had heterogeneous residuals. According to the results, models (11), (14), (13), (15) and (12) had a better fitness compared to other models. Among these models, model (11) was the best model for predicting total height of *Acer velutinum* trees in this region.

Keywords: height-diameter model; *Acer velutinum*; north of Iran

Covering an area of approximately 1.85 million ha, the Hyrcanian forests account for approximately 15% of the Iranian forests and 1.1% of the country's total area. These forests range from sea level up to an elevation of 2,800 m and comprise various forest types including no less than 80 woody species (trees and shrubs). Oriental beech (*Fagus orientalis* Lipsky), oak (*Quercus castaneifolia*), maple (*Acer velutinum*), hornbeam (*Carpinus betolus*) and alder (*Alnus subcordata*) are among the main tree species in these forests. The Hyrcanian forests have been forestland since the third geological era and are considered to belong to the oldest forests in the world (Sagheb-Talebi et al. 2004).

The relationship between tree height and diameter is an important element of forest structure (Wykoff et al. 1982; Van Deusen, Biging 1985; Ritchie, Hann 1986; Larsen, Hann 1987; Larsen 1994) that has long been used to describe the strategies of tree species among individuals at a particular point, to estimate timber volume and site index, and is also an important variable in growth and yield modelling (Parresol 1992). A number of tree height-diameter equations have been developed for various tree species (Colbert et al. 2002). In most of these models DBH is a predictor variable for estimating total tree height. Because diameters can be measured at low cost, and height measurements are considerably more difficult and costly to collect, many foresters only subsample total tree heights and do not measure heights at all (Krisnawati et al. 2010; Ahmadi et al. 2013).

Curtis (1967) examined many forms of height-diameter equations using linear regression techniques. Huang et al. (1992) selected the most appropriate height-diameter functions for major tree species. Zhang (1997) cross-validated six non-linear growth functions fitted to the tree height-diameter data of ten conifer species collected in the inland northwest of the United States and showed that the Schnute, Weibull, and Chapman-Richards functions were the best model. Fang and Bailey (1998) used 33 height-diameter models in tropical forests on Hainan Island in southern China. Peng et al. (2001) fitted six commonly used non-linear growth models and Sáncheza et al. (2003) used 26 linear and non-linear height-diameter functions. Lumbres et al. (2011) fitted and validated the height-diameter models for three species in South Korea and showed that the modified logistic and Lundqvist/Korf models performed best compared to the other models.

In order to determine the most appropriate relationship between the diameter and height of *Picea abies* L. in Kelardasht afforested area (in the north of Iran), Pourmajidian (1992), Siahipour et al. (2002), and Fallah (2009) fitted several non-linear models to *Picea abies* L. in Kelardasht afforestation data and introduced the most satisfactory model. Ahmadi et al. (2013) developed six non-linear growth functions to tree height-diameter data of oriental beech in the Hyrcanian mixed hardwood forests of Iran where each of the six models accounted for approximately 75% of total variation in height, and the Chapman-Richards functions provided the most satisfactory height predictions.

One of the important hardwoods in the Hyrcanian Forests is *Acer velutinum.* Because of the importance of *Acer velutinum* as one of the main species of the Hyrcanian Forests, this study aims to develop nonlinear height-diameter models for this species and to select the proper candidate model for further validation and volume estimation in this region.

MATERIAL AND METHODS

Study site. The study area is located in District one, two and three of Nav Asalem forest stands, in the southern Alborz Mountains, Guilan province, Iran. The forests of the study area are mixed and uneven-aged, dominated by *Acer velutinum* associated with *Fagus orientalis*, *Carpinus betulus*, *Alnus* spp., *Acer cappadocicum, Ulmus glabra, Fraxinus excelsior,* etc. These forests are managed as close-to-nature with single tree harvesting methods. For this study, five natural velvet maple forest stands were selected and studied. The study area is about 7.5 ha and extends from 520 to 1,150 m a.s.l. The study area is located between longitude 48°44'36'' and 48°52'27'' and latitude 37°37'23'' and 37°42'31''.

Mean annual rainfall is 1,286.5 mm with maximum and minimum falls in October and June, respectively. Temperatures vary from a mean monthly minimum of −19.5°C in January and February to a maximum of 30°C in June and July. Also, mean annual temperature is 8.5°C. This region has a cold and moderate climate according to the De Martonne climate index (Asadollahi 1987). The parent material is igneous rock (lime and acidic). Locally conglomerates and sandstones are also present. Most soils are sandy loam and clay loam and they are classified as Antisols, Inceptisols and Alphisols (Sheikholeslami 1998).

Methods. Referring to existing information and documents, and based on field inspections in the forest, the natural ranges of velvet maple were identified. Then, five intact or less intact natural sites were selected among the identified sites for this study. In each site, DBH and total height of all velvet maple trees were measured and recorded. Data were collected in the summer of 2012. A wide variety of height-diameter models have already been used for modelling the relationship between tree height and DBH.

Models. A complete list of selected height-diameter models used in the present study is given in Table 1. Overall, nineteen candidate models were considered.

Model performance criteria. There are several criteria for selecting the best regression model through the performance of a number of multiple measurements instead of single measurements, which is shown to be more objective and common (Aertsen et al. 2010; Ahmadi et al. 2013).

In the present study, various criteria were chosen and applied to evaluate the predictive performance of the models. These criteria include Akaike information criterion (AIC), Bayesian information criterion (BIC), root mean square error (RMSE), mean error (ME), and adjusted coefficient of determination (R^2_{adj}). The expressions have been summarized as the following equations (1–5):

$$\text{Adj. } R^2 = 1 - (1 - R^2) \times \frac{n-1}{n-p-1} \quad (1)$$

$$\text{ME} = \frac{\sum_{i=1}^{n}(\text{H}_i - \hat{H}_i)}{n} \quad (2)$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(\text{H}_i - \hat{H}_i)^2}{n}} \quad (3)$$

$$\text{AIC} = n \times \ln(\text{RMSE}) + 2P \quad (4)$$

$$\text{BIC} = n \times \ln(\text{RMSE}) + 2P \quad (5)$$

In the upper equations, H_i, $\hat{H}_i$, $\bar{\text{H}}$ and p stand for observed value, fitted value, mean of the observed values and number of parameters used in the model, respectively.

The NLS function in free statistical software packages R (R Development Core Team 2013) was used to fit 19 candidate models. For estimating the parameters, the initial values obtained by other authors in similar studies were used. The assumption of homoscedasticity was investigated by plotting the fitted values versus the residuals (Ritz, Streibig 2008). For model validation, the *k*-fold cross validation method with $k = 10$ has been used. The *k* taking the value 10 has been the most common practice (Olson, Delen 2008; Diamantopoulou, Özçelik 2012). In this method the data is randomly partitioned into *k* equal subsamples, one of which is considered as the validation data for evaluating the model, and the other subsamples ($k - 1$) are used for training. 10-fold cross-validation was performed using the cvTools-package in R software (R Development Core Team 2013).

RESULTS

Fitting of nineteen height-diameter models using nonlinear least square regression showed that all of the parameters in models were significant ($P < 0.01$). Estimated parameters of nineteen height growth functions for velvet maple are given in Tables 2 and 3.

Table 1. Non-linear height-diameter models selected for the present study

Model		References
1	$h = 1.3 + a \times \text{DBH}^b$	Stoffels, van Soeset (1953); Stage (1975); Schreuder et al. (1979)
2	$h = 1.3 + e^{a + b/(\text{DBH} + 1)}$	Wykoff et al. (1982)
3	$h = 1.3 + a \times \text{DBH}/b + \text{DBH}$	Bates, Watts (1980); Ratkowsky (1990)
4	$h = 1.3 + a \times (1 - e^{-b \times \text{DBH}})$	Meyer (1940); Farr et al. (1989); Moffat et al. (1991)
5	$h = 1.3 + \text{DBH}^2/(a + b \times \text{DBH})^2$	Loetsch et al. (1973)
6	$h = 1.3 + a \times e^{b/\text{DBH}}$	Burkhart, Strub (1974); Burk, Burkhart (1984); Buford (1986)
7	$h = 1.3 + 10^a + \text{DBH}^b$	Larson (1986)
8	$h = 1.3 + (a \times \text{DBH})/(\text{DBH} + 1) + b \times \text{DBH}$	Watts (1983)
9	$h = 1.3 + a \times (\text{DBH}/1 + \text{DBH})^b$	Curtis (1967); Prodan (1968)
10	$h = 1.3 + a \times e^{(-b \times \text{DBH}^c)}$	Stage (1963); Zeide (1989)
11	$h = 1.3 + a/(1 + b \times e^{-c \times \text{DBH}})$	Pearl, Reed (1920)
12	$h = 1.3 + a \times (1 - e^{-b \times \text{DBH}})^c$	Richards (1959)
13	$h = 1.3 + a \times (1 - e^{-b \times \text{DBH}^c})$	Yang et al. (1978)
14	$h = 1.3 + a \times e^{-b \times e^{-c \times \text{DBH}}}$	Winsor (1932)
15	$h = (\text{H}_1)^b + ((\text{H}_2)^b - (\text{H}_1)^b) \times (1 - e^{-a \times \text{DBH} - \text{DBH}_{\text{min}}})/(1 - e^{(-\text{DBH}_{\text{max}} - \text{DBH}_{\text{min}})} \times a)$	Schnute (1981)
16	$h = 1.3 + (\text{DBH}^2/\ a + b \times \text{DBH} + c \times \text{DBH}^2)$	Curtis (1967); Prodan (1968)
17	$h = 1.3 + a \times \text{DBH}^{b \times \text{DBH}^{-c}}$	Sibbesen (1981)
18	$h = 1.3 + a \times e^{b/(\text{DBH} + c)}$	Ratkowsky (1990)
19	$h = 1.3 + a/(1 + b^{-1} \times \text{DBH}^{-c})$	Ratkowsky, Reedy (1986)

Table 2. Parameter estimates for height-diameter models (1–9) for *Acer velutinum* trees

Parameter	Model								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	4.72	3.813	52.295	40.36	2.047	43.805	0.674	15.54	44.075
B	0.442	–19.187	43.242	0.024	0.145	–19.202	0.442	0.203	19.854
SSE	3.240	3.002	2.965	2.856	2.946	3.022	3.240	3.628	3.012

Table 3. Parameter estimates for height-diameter models (10–19) for *Acer velutinum* trees

Parameter	Model									
	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
A	102.68	40.531	47.773	49.044	41.625	0.05	–2.488	69.832	54.411	62.846
B	–4.984	2.853	0.011	0.047	1.537	1.835	0.962	–4.334	–48.291	0.061
C	–0.344	0.033	0.611	0.728	0.025	10.000	0.018	0.752	20.779	0.819
SSE	2.985	2.684	2.847	2.832	2.756	2.841	2.952	2.963	2.938	2.920

The results of goodness of fit for the calibration and *k*-fold validation are shown in Table 4. The performance criteria (RMSE, ME, AIC, R^2_{adj} and BIC) are given in Table 4 for both calibration and 10-fold validation. Models with the lowest RMSE, ME, BIC and AIC values and the adjusted R^2 closest to unity are known to perform best (Ahmadi et al. 2013). R^2_{adj} ranged from 0.743 (model 8) to 0.8592 (model 11) and RMSE from 2.6983 (model 11) to 10.1897 (model 9). The range of ME among the models is from –7.0787 (model 9) up to 0.063 m (model 7). By considering the AIC for each model it is evident that model 11 and 9 have the lowest and highest values, respectively. Values of BIC for validation data are presented in Table 4. Fig. 1 illustrates scatter plots of the tree diameter and height data for all data sets.

Plotting the residuals showed that for all these models the residuals were randomly distributed while the models had heterogeneous residuals. Fig. 2 shows plots of predicted height versus residuals and predicted height versus observed height.

Table 4. Performance criteria of 19 height-diameter models for the validation and calibration data

Model	Performance criteria									
	calibration					validation				
	Adj. R^2	RMSE	BIC	ME	AIC	Adj. R^2	RMSE	BIC	ME	AIC
1	0.7999	3.2271	299	0.0571	292	0.7960	3.2571	301	0.063	294
2	0.8281	2.9903	280	–0.0567	273	0.8255	3.0139	282	–0.0569	275
3	0.8320	2.9538	277	0.0275	270	0.8291	2.979	279	0.0311	272
4	0.8441	2.8447	268	0.0147	261	0.8414	2.8688	270	0.0179	263
5	0.8342	2.9343	275	–0.0272	268	0.8315	2.9579	277	–0.0254	270
6	0.8260	3.0106	282	–0.0651	275	0.8234	3.0343	284	–0.0655	277
7	0.7999	3.2271	299	0.0571	292	0.7960	3.2571	301	0.063	294
8	0.7483	3.6141	327	0.0131	320	0.7430	3.6515	329	0.0221	322
9	0.8271	3.0002	281	–0.0608	274	0.7858	10.1897	582	–7.0787	575
10	0.8297	2.9672	284	–0.0227	273	0.8257	3.0018	286	–0.0214	276
11	0.8624	2.6677	257	–0.0241	247	0.8592	2.6983	260	–0.0223	250
12	0.8452	2.83	272	–0.0316	261	0.8417	2.8622	275	–0.0302	264
13	0.8470	2.8154	271	–0.0515	260	0.8435	2.8486	274	–0.0503	263
14	0.8548	2.7395	264	–0.0182	253	0.8515	2.7703	267	–0.0165	255
15	0.8460	2.824	271	–0.0429	261	0.8424	2.8568	274	–0.0415	264
16	0.8334	2.9342	281	–0.0261	270	0.8290	2.9741	284	–0.0252	274
17	0.8322	2.9452	282	–0.0194	271	0.8282	2.9795	285	–0.018	274
18	0.8349	2.9206	280	–0.0079	269	0.8313	2.9522	282	–0.0063	272
19	0.8372	2.9024	278	–0.0387	268	0.8335	2.936	281	–0.0372	270

AIC – Akaike information criterion, BIC – Bayesian information criterion, RMSE – root mean square error, ME – mean error, R^2_{adj} – adjusted coefficient of determination

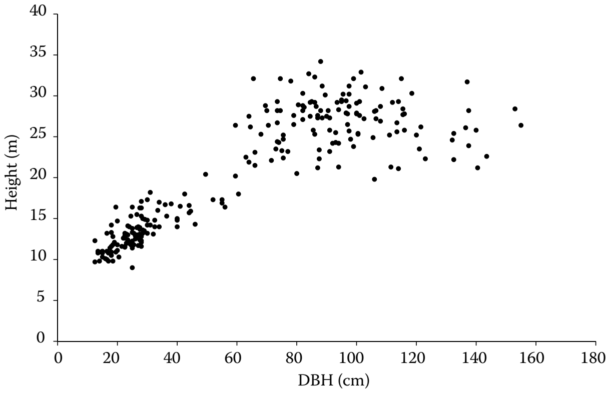

Fig. 1. Total height plotted against diameter at breast height (DBH) for *Acer velutinum* trees

DISCUSSION AND CONCLUSIONS

Tree height is fundamental for developing growth and yield models in forest stands and is also an important variable in volume estimation and biomass calculation (Diamantopoulou, Özçelik 2012). The development of simple and accurate height diameter models enables forest managers to make the reliable prediction of tree height in forests (Calama, Montero 2004). To achieve this, nineteen nonlinear models were used in this study. Selection of the best and most accurate models should be based on appropriate criteria (Zhang 1997) such as AIC, BIC, RMSE, ME and R^2_{adj}, which were used in this study. Based on 5 statistical criteria we can suggest the best model for the height growth prediction of *Acer velutinum* (northern Iran). The first model was ranked based on the performance criteria (Table 5). According to the results models (11), (14), (13), (15) and (12) had a better fitness compared to the other models. This result is consistent with the findings of Zhang (1997) in the United States, and Peng et al. (2001) in Canada. Based on the development data, among these models, model (11), suggested by Pearl and Reed (1920), was the best model for predicting total height of *Acer*

Table 5. Summary of ranks of model performance for velvet maple

Model performance	Calibration data	Validation data
Adj. R^2	11, 14, 13, 15, 12	11, 14, 13, 15, 12
RMSE	11, 14, 13, 15, 12	11, 14, 13, 15, 12
AIC	11, 14, 4, 13, 12	11, 14, 13, 4, 12
BIC	11, 14, 4, 13, 15	11, 14, 4, 13, 15

AIC – Akaike information criterion, BIC – Bayesian information criterion, RMSE – root mean square error, ME – mean error, R^2_{adj} – adjusted coefficient of determination

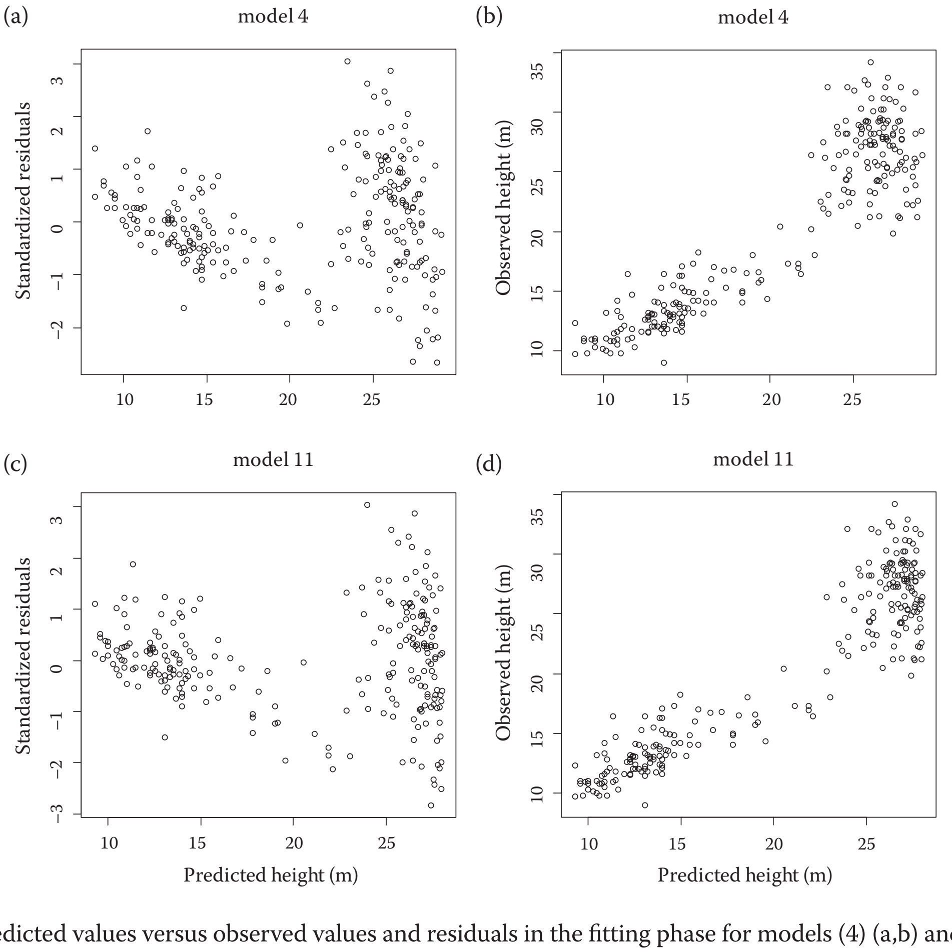

Fig. 2. Plot of predicted values versus observed values and residuals in the fitting phase for models (4) (a,b) and (11) (c,d)

velutinum trees in this region. However, the performance of other models, such as the Schnute one (model 15), was also very good. These models have been shown to be very flexible and have been used extensively in growth and yield studies for describing height-age, diameter-age, and volume-age relationships (Somers, Farrar 1991). However, the relationship between diameter and height varies within a region, depending on local environmental conditions, and also varies within a geographic region (Özçelik et al. 2014). It is recommended that height-diameter models would be surveyed by ecoregion for reflecting the regional differences. The ecoregion-based models can provide useful tools to forest resource managers in forest management practices and decision making.

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