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A fixed count sampling estimator of stem density based on a survival function

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ABSTRACT: In fixed count sampling (FCS) a fixed number (k) of observations is made at n randomly selected sample locations. For estimation of stem density, the distance from a random sample location to the k nearest trees was measured. It is known that practical FCS estimators of stem density are biased. With the objective of reducing bias in FCS estimators of stem density, a new estimator derived from a survival function with distance acting as time was presented. To allow for spatial heterogeneity in stem density, the survival function includes shared frailty. Encouraging results with $k = 6$ in terms of bias, root mean squared error (RMSE), and coverage of nominal 95% confidence intervals were obtained in an extensive testing with simulated random sampling from 54 actual and four simulated spatial point patterns of tree locations. Sample sizes were 9, 15, and 30, with 1200 replications per setting. The performance across sites of the new FCS estimator was variable but almost paralleled that of a design-based estimator with fixed area plots. Users of the new FCS estimator can expect an absolute relative bias and a root mean squared error that are 1% greater than for sampling with fixed area plots holding an average of k trees. The chance of a smaller RMSE with the proposed estimator was estimated at 0.44.

Keywords: bias; coverage; forest inventory; spatial point pattern; standard error; root mean squared error

In forestry application of fixed count sampling (FCS) a fixed number (k) of trees closest to each of n randomly selected sample locations is observed and recorded for attributes (Y) of interest (Morisita 1954; Shanks 1954; Persson 1964; Payandeh, Ek 1986). Density estimators derived from FCS are biased because the inclusion area of the k trees depends on the sample location (Moore 1954; Eberhardt 1967; Ohtomo 1971; Pollard 1971; Cox 1976). The bias can be estimated analytically if and only if the spatial point pattern is consistent with a realization from Poisson process (Moore 1954; Pollard 1971) or a negative binomial distribution (Eberhardt 1967).

In difficult terrain and in populations with a high and variable local density of elements, the FCS can be faster and more expedient than sampling with fixed area plots (Cox 1976; McNeill et al. 1977; Chen, Tsai 1980; Patil et al. 1982; Roser et al. 1984; Delince 1986; Jianrang 1988; Lessard et al. 1994; Barbour, Gerritsen 1996; Lynch, Rusydi 1999; Doberstein et al. 2000; Lessard et al. 2002; Picard et al. 2005; Kleinn, Vilčko 2006b; Haxtema et al. 2012).

Efforts to address the bias problem in FCS have been considerable, especially in the context of forest management inventories, and they reflect a general interest in this sampling scheme (Persson 1964; Cox 1976; Kleinn, Vilčko 2006a, b; Magnussen et al. 2008a; Kronenfeld 2009; Nothdurft et al. 2010; Magnussen et al. 2011, 2012a, b, 2014). Design-unbiased FCS density estimators are possible (Kleinn, Vilčko 2006a; Fehrmann et al. 2011) but remain impractical.

As a result we now have several alternative FCS estimators of stem density, each with a distinct performance profile in terms of bias and root mean squared error. Although individual studies from a few select sites or point patterns have succeeded in a significant reduction of the bias problem, it should be recognized that the bias problem often reappears when sampling from a different set of spatial point patterns (Magnussen et al. 2011; Magnussen 2012b). The seemingly erratic performance of some FCS density estimators deemed robust and suitable for a particular set of point patterns (Delince 1986; Kleinn, Vilčko 2006b;

Kronenfeld 2009) illustrates the complexity and difficulty of reducing the bias problem in general.

Five recently proposed and broadly applicable FCS stem density estimators (Nothdurft et al. 2010; Magnussen et al. 2011; Magnussen 2012a, b; Magnussen 2014) appear to have reduced the bias to a range (–6 to +6%) where it is likely to be of limited practical concern in FCS with k-values of 4, 5, or 6, and a relatively small sample size ($n \leq 50$). However, two of the five estimators are of limited practical utility as they incur a non-trivial computational burden (Nothdurft et al. 2010; Magnussen 2012b). Although the bias problem was effectively addressed in the remaining three FCS density estimators (Magnussen et al. 2011; Magnussen 2012a, 2014), their performance in terms of root mean squared error (RMSE) and coverage rates of nominal 95% confidence intervals (cCI95) still lags behind the performance with a comparable fixed-area plot design.

In a quest to further reduce bias and improve RMSE and cCI95 of FCS density estimators, this study proposes a new estimator of stem density derived from a parametric survival function (Hosmer, Lemeshow 1999) whereby the distance (d_{ij}) from the i^{th} sample location (i = 1, ..., n) to the j nearest tree (j = 1, ..., k) serves as 'survival time'. Estimation of a parametric survival function provides a model for the probability of observing a distance to the 1st, 2nd,...., k^{th} nearest tree equal to or less than some user-specified distance. To compute an expected 'survival' distance and subsequently a stem density, one also needs an estimate of the probability distribution function of distances in the sampled population. A complicating factor, however, is the common phenomenon of local variation in stem density (Clayton, Cox 1986; Picard et al. 2005; Kronenfeld 2009; Nothdurft et al. 2010). In the context of FCS, a local variation in stem density becomes apparent when the variance in the distance to the k^{th} nearest tree is larger than in a forest with a Poisson distribution of stem locations (Thompson 1956). If the k-nearest trees are considered as a cluster, a local variation in stem density will generate a positive intra-cluster correlation among the k distances observed at a single sample location. The local variation in stem density can be captured in a survival function by adding a random "cluster" effect called frailty (Wienke 2010). Since the effect is viewed as shared among the k trees, it is called a shared frailty.

The proposed FCS estimator of stem density is tested on a set of 54 actual and four simulated spatial point patterns of forest tree locations. The same point patterns have been used in previous assessments of novel FCS estimators of stem density (Magnussen et al. 2011, 2012a; Magnussen 2014). A wide variation in the test patterns allows a generalization of the performance assessment beyond a forest management inventory.

MATERIAL AND METHODS

The proposed FCS estimator of stem density $\left(\lambda_S^{(k)}\right)$ is designed for simple random sampling (SRS) of n sample locations in a finite-area population with a countable finite number of trees. At each of the n sample locations, the distances from the sample location to the k nearest trees are measured. The proposed estimator takes the same form as a maximum likelihood estimator (MLE) of stem density in a Poisson point pattern (Pollard 1971). However, model-dependent predictions are used in place of observed distances. The proposed estimator is given in Equation 1:

$$\hat{\lambda}_S^{(k)} = \frac{k - \tilde{c}}{\pi E_n\left[\tilde{d}_k^2\right]} = \frac{k - \tilde{c}}{\pi\left(E_n\left[\tilde{d}_k\right]^2 + \tilde{V}\left(d_k\right)\right)} \quad (1)$$

where:

k – fixed number of trees to include in the sample at a random sample point,

$\tilde{c}$ – model-dependent bias correction term,

E_n – expectation over the sample,

$\tilde{d}_{ik}$ – model-dependent prediction of the expected distance from a random sample location (i) to the k-nearest element,

$\tilde{V}\left(d_k\right)$ – model-dependent prediction of the variance in distances d_k to the k^{th} nearest tree.

The rationale for using model-dependent predictions in Eq. (1) as opposed to the observed distance data rests with an expectation that a modelling of distances by a parametric survival function with a shared frailty will reduce an otherwise expected bias in the MLE estimator for a Poisson forest.

The model-dependent predictions and bias-correction terms in Eq. (1) were obtained by assuming that one minus the cumulative distribution function of distances $F_{ij}(d)$ from a random sample location i, to the j^{th} nearest tree follows a parametric survival function with a shared frailty term (Gutierrez 2002).

The model predictions in Eq. (1) depend on: (i) the choice of the parametric survival function; and (ii) the choice of a distribution for the assumed shared frailty. In practical application an analyst may choose to optimize these choices via a maximum likelihood estimation of model parameters. Results in this study are based on a Weibull distribution function for distances ($F(d) = 1 - \exp[-\tau d^{\phi}]$), and a gamma distribution for the shared frailty term (α). This choice was based on promising results from simulated sampling in four test patterns (see the section on test sites) and choices available in the streg procedure of the STATA®-13 software (Stata LP, College Station, Texas) (GUTIERREZ 2002). With these choices and the assumption that the Weibull parameter τ is an exponential function of the distance order (i.e. j) we have (Eq. 2):

$$F_{ij}(d^*) = \Pr(d_{ij} \leq d^*) = (1 - \exp[-\tau_j (d^*)^{\phi}]^{i}) \quad (2)$$
with $\tau_j = \exp[\beta_0 + \beta_1 \log(j)]$ and $\alpha_1 \sim \text{Gamma}(\theta^{-1}, \theta)$

where d^* is a user specified distance. Note how the random frailty term is specific to a location (i) and therefore shared among the distances to the 1, ..., k^{th} tree at this location. The parameters $\Psi = \{\phi, \beta_0, \beta_1, \theta\}$ in Eq. (2) were estimated via the method of maximum likelihood (ML) (CLEVES et al. 2004) using all $n \times k$ observed distances d_{ij} in the likelihood under the assumption of conditional independence of distances d_{ij}, $j = 1, ..., k$ given Ψ and θ. It is important to note that the survival function is fitted to all observed distances. Hence, the expected distance to the k^{th} nearest tree cannot be derived directly from the survival function, it has to be weighted by a probability density function of distances to the k^{th} nearest tree.

Alternative models for τ_j in Eq. (2) with squared and inverse transforms of j within the exponent were also explored but without success. A simpler approach using a hazard function in place of a survivor function in Eq. (2) that was also tried, gave a much more variable performance.

As mentioned above, the estimated distribution function of distances is estimated jointly for all $n \times k$ distances. A model-dependent prediction of the expected distance $\tilde{d}_k$ appearing in the estimator $\hat{\lambda}_S^{(k)}$ (see Eq. 1) must therefore be calibrated to the distribution of d_k. For the spatial point patterns used in this study, a gamma distribution with parameters $\{\gamma_k, \eta_k\}$ provides (consistently) a good fit to the distribution of d_k (MAGNUSSEN 2012a). As k increases, the distribution approaches that of a standard gamma distribution $(\text{ie. } \gamma_k \rightarrow 1, \text{for } k \rightarrow \infty)$. With these choices the expected distance was computed as (Eq. 3):

$$\tilde{d}_k = \frac{\int_{d_{min}}^{d_{max}} d_k F'\left(d_k \mid \hat{\Psi}\right) g\left(d_k \mid \hat{\gamma}_k, \hat{\eta}_k\right) dd_k}{1 - \int_{d_{min}}^{d_{max}} F'\left(d_k \mid \hat{\Psi}\right) g\left(d_k \mid \hat{\gamma}_k, \hat{\eta}_k\right) dd_k} \quad (3)$$

where $F'(d_k \mid \Psi)$ is the probability distribution function of d_k, viz. the derivative of the survival function F. In the integrations, the lower 1% and upper 99% quantiles of the gamma distribution were used as integral limits (d_{min}, d_{max}) in an effort to reflect a finite domain of tree sizes. The same integration procedure was used to compute the expected variance of $\tilde{d}_k$, viz. $\tilde{V}(d_k)$. Specifically, the variance was computed as the expected value of $\left(d_k - \tilde{d}_k\right)^2$ over the distributions of d_k using the same integration limits as in Eq. (3).

The bias correction term $\tilde{c}$ in Eq. (4) reflects a long-standing recognition of the need, in FCS estimators of density, to correct the count (k) of elements within the circle with a radius d_k (MOORE 1954; MORISITA 1954; PERSSON 1964; POLLARD 1971; PATIL et al. 1982; DELINCE 1986). POLLARD (1971) derived a bias correction factor of $1/n$ for the Poisson forest. MOORE (1954) suggested a multiplicative bias correction of $(k-1)/k$. KLEINN and VILČKO (2006b) suggested a bias reduction by using the geometric mean of the distances to the k^{th} and $(k-1)^{th}$ tree. However, these corrections are inefficient beyond a few select types of point patterns (KLEINN, VILČKO 2006b; MAGNUSSEN et al. 2008). Instead, the proposed model-dependent bias correction term $\tilde{c}$ is the binomial variance of the 'survival' probability of the predicted distance $\tilde{d}_k$. That is Equation (4):

$$\tilde{c} = \Pr(d_{ik} \leq \tilde{d}_k)\left(1 - \Pr(d_{ik} \leq \tilde{d}_k)\right)$$
$$\text{with } \Pr(d_{ik} \leq \tilde{d}_k) = 1 - \hat{\theta} \log\left(1 - \exp\left(-\hat{\tau}_j \tilde{d}_k^{\phi}\right)\right) \quad (4)$$

The bias correction in (4) is similar to a bias correction of the expectation of a log-transformed variable (BASKERVILLE 1972; SNOWDON 1991). The random location effect (α_i) acts akin to a cluster effect (BANERJEE et al. 2003). If the local density varies at random across a surveyed population, distances to the 1st, ..., k^{th} element at a single sample location will be less variable than if the k ordered distances came from k randomly selected locations. Both clustered and over-dispersed point patterns are likely to exhibit significant location effects ($\theta > 1$). Conversely, a quasi-regular point pattern would exhibit less variation than in a random point pattern ($\theta < 1$). As θ increases, the variance in distances to the j^{th} nearest element also increases, and the distribution of d_{ij} becomes increasingly right-skewed.

During the computations of $\lambda_S^{(k)}$ for the four simulated spatial test patterns (see the section on test sites) it became clear that very small and very large estimates of θ led to extreme and implausible estimates of point density. It was therefore decided to trim $\hat{\theta}$ to the interval [0.01, 6.0]. Less than 1.5% of 208, 800 computed density estimates were affected by the trimming.

A variance estimator for $\lambda_S^{(k)}$ was derived via the delta technique (Kotz, Johnson 1988; Oehlert 1992) applied to the estimator in Eq. (1) with $\tilde{d}_k^2 + \tilde{V}\left(d_k\right)$ replaced by the expectation $E\left(\tilde{d}_k^2\right)$ computed over the gamma distribution $g\left(\tilde{d}_k \mid \hat{\gamma}_k, \hat{\eta}_k\right)$. With the delta technique the variance of $\lambda_S^{(k)}$ is estimated as $\left(\partial\lambda_S^{(k)} / \partial E\left(\tilde{d}_k^2\right)\right)^2 \times \text{variance}\left[E\left(\tilde{d}_k^2\right)\right]$. Although good results were obtained with this estimator – in terms of matching the empirical variance and coverage of 95% confidence intervals – it did fail on six sites with a relatively low stem density and a visible clustering of tree locations. The reason for the failure was the exponential increase in higher moments in a right-skewed distribution of distances. Instead, we modified the variance estimator obtained from the delta technique and propose the following more robust and conservative estimator of variance

$$\tilde{V}\left(\hat{\lambda}_S^{(k)}\right) = \frac{4\left(k - \tilde{c}\right)^2}{\pi^2\left(E\left(\tilde{d}_k^6\right)\right)} \frac{\tilde{V}\left(d_k\right)}{k-1} \qquad (5)$$

The term $E\left(d_k^6\right)$ in Eq. (5) was computed using the same integration employed for computing an expectation in (3).

The computations behind $\lambda_S^{(k)}$ and $\tilde{V}\left(\hat{\lambda}_S^{(k)}\right)$ may seem daunting; yet attempts at simplifications (e.g. dropping the gamma distribution of distances, or dispensing with the random location effects) led to a significant drop in performance (bias, RMSE, and cCI95). For a single sample with n = 15, the time required to estimate the MLE of $\hat{\Psi}$, and the gamma distribution parameters with the software package STATA®-13 (Cleves et al. 2004) and a typical desktop computer is 30–50 s. Generally, computing times were proportional to the estimate of θ.

Assessment of performance. The performance of the proposed FCS estimator in Eq. (1) was assessed in simulated random sampling in 58 spatial point patterns (viz. sites). Performance criteria were: bias; root mean squared error (RMSE); how well the replication average of the analytical estimator of variance in Eq. (5) tracks (across patterns) the corresponding empirical variance in replicated estimates of density; and the achieved coverage of nominal 95% confidence intervals (cCI95) for the true point density. Bias, for a given test pattern and sample size n, was estimated as the difference between the mean of 1,200 estimates of density $\left(\hat{\bar{\lambda}}_S\right)$ and the known density λ_{site}. Estimates of bias are given in percent of λ_{site} in order to facilitate comparisons across sites (patterns). A RMSE was computed as $\sqrt{(1{,}200 - 1)^{-1}\sum_{rep=1}^{1200}\left(\hat{\lambda}_{S,rep} - \lambda_{site}\right)^2}$. To facilitate an among-site comparison, the relative root mean squared error (RRMSE) – computed as RMSE divided by λ_{site} – is reported. Linear regression analysis was employed to assess how well the mean of replicated analytical estimates of variance obtained from (5) tracked the corresponding empirical estimate of variance across the 58 test patterns (sites). The achieved coverage rate of a nominal 95% confidence interval was computed as the proportion of computed normal theory confidence intervals (Casella, Berger 2002) which included λ_{site}.

Results obtained with $\hat{\lambda}_S$ were compared to results with a design-unbiased fixed-area density estimator λ_{FIX} as well as to results with two FCS estimators of density previously proposed by the author. The fixed-area plots used with λ_{FIX} were circular with a radius r_k that, on a given test pattern (site), included an average of k elements. Hence the expected density is $k\pi^{-1}r_k^{-2}$. Because λ_{FIX} is only nearly asymptotically $\left(n \to \infty\right)$ unbiased (Gregoire, Valentine 2008; Mandallaz 2008), we computed estimates of bias and RMSE as done for $\hat{\lambda}_S$. This ensures a fair comparison, since our sampling protocol and number of replications (1,200) did not attest a zero bias in $\hat{\lambda}_{FIX}$. The two alternative FCS estimators used in the comparison are called $\hat{\lambda}_{V1}$, $\hat{\lambda}_{V2}$ where the subscript V stands for "virtual fixed area plot" (Magnussen 2012a); the estimator $\hat{\lambda}_{V2}$ was intended as a robust version of $\hat{\lambda}_{V1}$ (Magnussen 2014). Based on results from an intensive testing of both $\hat{\lambda}_{V1}$ and $\hat{\lambda}_{V2}$ it does not seem presumptuous to consider them as two well performing – in terms of bias and RMSE – FCS estimators of density. Although the FCS reconstruction density estimator λ_{RDE} by Nothdurft et al. (2010) is asymptotically unbiased, and therefore a natural choice as the currently best FCS estimator of density; computational complexities deter use in practice and especially for this study with a large number of Monte Carlo simulations of simple random sampling. In terms of RMSE the estimator $\hat{\lambda}_{V2}$ holds an edge over λ_{RDE} (Magnussen 2014). Computational complexities equally excluded an otherwise attractive FCS estimator with evidence-based (Akaike's Information Criterion) averaging of maximum-likelihood based estimators of density based on 16 different models for the spatial distribution of distances to the k^{th} nearest element (Magnussen 2012b).

The $\hat{\lambda}_{V1}$ estimator computes density on the basis of a virtual fixed area circular plot with a radius r_v to be computed from the observed distances. A generic recursive algorithm is used to predict the distance to the $k+m$ nearest element from distances to the $k+m-1$ and $k+m-2$ nearest elements ($k > 2$, $m = 1, \ldots, M$). With this algorithm, a prediction of the number of elements in a virtual plot with a dis-

Stand 17

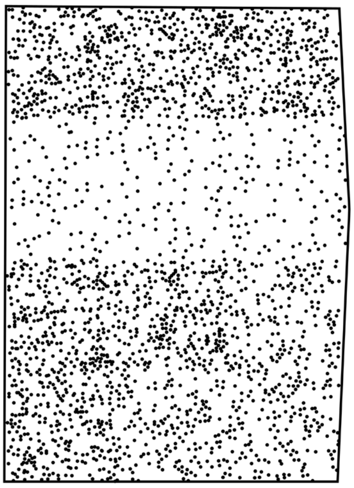

Fig. 1. Stem locations in a 6.4 ha old spruce stand in Stand 17 (Schöpfer 1967), stem density 370 ha (site 28)

tance to the k^{th} element less than r_v is obtained with a minimum of computational effort (Magnussen 2012a). In $\hat{\lambda}_{V2}$ predictions of distances to the $(k+1)^{th}$, $(k+2)^{th}$, …, $(k+m)^{th}$ nearest element are based on a nonlinear model of distance ratios $d_{k+m} \times d_{k+m-1}^{-1}$ ($k > 2$, $m = 1, …, M$) derived from a mixture of observed distances and distances predicted with the recursive algorithm used in $\hat{\lambda}_{V1}$. The use of actual and predicted distances to fit the nonlinear (Pareto-type) model of distance ratios was assumed to bestow 'robustness' to $\hat{\lambda}_{V2}$. The performance profile of $\hat{\lambda}_{V1}$ and $\hat{\lambda}_{V2}$ – across the same 58 spatial point patterns used in this study – was, from a practical perspective, however, quite similar.

Simulated fixed count sampling. Fixed count sampling from a finite area of spatial point locations was simulated with a simple random sampling without replacement (SRS) of n out of N possible locations on a regular 1 m × 1 m grid suspended over the area of interest. At each sample location, the distance (d in m) to the nearest k points (elements) is measured. From a sample of $n \times k$ distances the density $\hat{\lambda}_S$ (elements·m^{-2}) was estimated as per (1) subsequent to a MLE of the parameters $\hat{\Psi}$ in the survival function, and the two parameters $(\hat{\gamma}_k, \hat{\eta}_k)$ for the assumed gamma distribution of distances. A model-dependent estimate of variance in $\hat{\lambda}_S$ was obtained via Eq. (5), and a normal-theory 95% confidence interval was computed by standard techniques (Casella, Berger 2002). The process of sampling and estimation was repeated 1,200 times for each sample design and test site (1, …, 58).

Sampling designs were limited to sample sizes n = 9, 15, and 30. The value of k was fixed at 6. Studies with spatial point patterns of tree locations (Lynch, Rusydi 1999; Lessard et al. 2002; Lynch, Wittwer 2003; Kleinn, Vilčko 2006b; Magnussen 2012b) confirm that FCS designs with $k < 6$ and $n < 9$ frequently generate unacceptable levels of bias (>10%) and unattractive estimates of errors.

To mitigate edge effects in sampling from a finite area (Gregoire, Valentine 2008), a selected sample location within a distance of r_k from the border of a test site was replaced by a randomly selected location from the available $N - n$ choices (Magnussen et al. 2011). This ensures a fixed sample size of n. In practice edge effects are mitigated by the 'mirage boundary' technique (Lynch 2012).

Test point patterns. Testing of λ_S was done with simulated random sampling from 54 actual and four simulated point patterns. The four simulated point patterns ("random, Matérn, quasi-regular, and Strauss") served the model development. The remaining 54 represent locations of trees (elements) in forest sites (stands). Point densities in the test set varied from 0.0046 to 0.7361 elements·m^{-2}, and the patterns ranged from quasi-regular (plantations) over random (Poisson) to strongly clustered. A total of 35 patterns were not statistically different from complete spatial randomness (*csr*, Illian et al. 2008), and 23 indicated a significant departure from *csr* (Kolmogorov-Smirnov test applied to the distribution of the distances to the k nearest element, Conover 1980). Further details are provided in (Magnussen 2012b). Nine contrasting point patterns have been displayed in Magnussen et al. (2011). Six additional patterns were displayed in (Nothdurft et al. 2010). Additional four of the 24 non-random patterns can be found in Magnussen (2014). Here the four patterns that gave rise to the largest absolute bias with $\hat{\lambda}_S$ in sampling with n = 15 are shown in Figs 1 to 4. It is immediately apparent why sample-based density estimation is a chal-

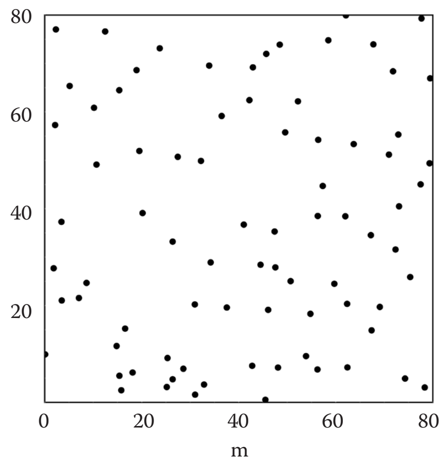

Fig. 2. Stem map of oak in a stand with a mixture of oak and beech (Manderscheid-198, Rheinland-Pfalz (Ger.), Pommerening 2002), stem density 125 ha (site 15)

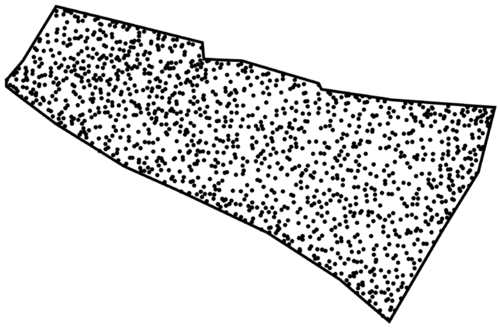

Fig. 3. Stem locations in a 3.1 ha stand with beech and shade tolerant hardwoods in Germany in stand 84(SCHÖPFER 1967), stem density 441 ha (site 51)

lenge in these examples. A test of first-order stationarity in density (ZHANG, ZHOU 2014) was rejected ($P < 0.05$) in each of the four examples.

RESULTS

We report results from all 58 point patterns sites without a distinction between the four simulated patterns used for model building and the 54 actual test patterns. There was no evidence to suggest a material difference between results from the two groups. Sampling with $n = 15$ is in focus, whereas results for $n = 9$, and $n = 30$ are summarily related to the former.

Bias

Over all test patterns and $n = 15$, the average estimate of bias in $\hat{\lambda}_S$ was –1.0%. Sampling with fixed area plots achieved, as expected, the lowest estimate of average bias (–0.5%). Comparable results for $\hat{\lambda}_{V1}$ and $\hat{\lambda}_{V2}$ were estimated at –1.0 and 0.4%. As seen in Fig. 5, the range in site-specific estimates of bias is wide for all four density estimators. To wit: a range from –17 to 11% in $\hat{\lambda}_{FIX}$; from –12 to 12% in $\hat{\lambda}_S$; from –18 to 9% in $\hat{\lambda}_{V1}$; and from –14 to 15% in $\hat{\lambda}_{V2}$. Centred intervals including 52 of the 58 point patterns (90%) were approximately 25% shorter. In terms of site-specific estimates of absolute bias, $\hat{\lambda}_{FIX}$ was again the best (mean 3.3%) followed by nearly equal estimates (3.9–4.1%) for $\hat{\lambda}_S$, $\hat{\lambda}_{V1}$ and $\hat{\lambda}_{V2}$. Density estimates with $\hat{\lambda}_{FIX}$ were the least biased in 23 cases but also the most biased in 7 cases. Corresponding figures were 6 and 17 for $\hat{\lambda}_S$, 13 and 13 for $\hat{\lambda}_{V1}$ and 16 and 21 for $\hat{\lambda}_{V2}$. Estimates of absolute bias were strongly correlated among the four estimators ($0.92 \le \rho \le 0.95$) indicating a great deal of parallelism in relative performance. The strong correlation is also indicated in Fig. 5, where the four estimator-specific scatter in site estimates of bias displays a considerable degree of resemblance. The four patterns with the highest levels of estimated bias with $\hat{\lambda}_S$ (Figs 1–4) were also included in the five patterns with the highest estimate of bias in $\hat{\lambda}_{FIX}$, $\hat{\lambda}_{V1}$ and $\hat{\lambda}_{V2}$.

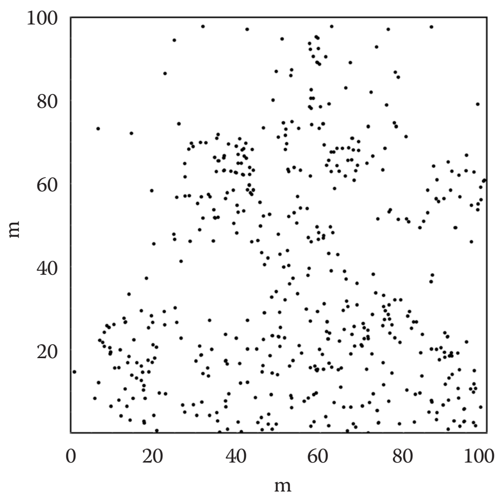

Fig. 4. Stem locations of maple in a natural stand of maple and hickory in East Lansing, MI, USA (HATCH et al. 1975), stem density 514 ha (site 11)

Estimates of bias in sampling with $n = 30$ were virtually identical to those for $n = 15$. Differences were less than ± 0.3% and within the range of Monte-Carlo errors. Sampling with $n = 9$, however, resulted in a minor (2.4%) increase in the average estimate of absolute bias in the three FCS density estimators, and a slight (1%) increase in the range of estimates of relative bias in $\hat{\lambda}_{FIX}$. No other result of practical relevance emerged from lowering the sample size from 15 to 9.

The estimates of bias in $\hat{\lambda}_{FIX}$, $\hat{\lambda}_{V1}$ and $\hat{\lambda}_{V2}$ are higher than those previously reported by MAGNUSSEN (2012a, 2014). Previous studies considered the average replicate value of $\hat{\lambda}_{FIX}$ as the actual true density (bias = 0). In this study we recognize that in patterns with a spatially varying density, the required number of replications needed to ascertain that the bias in $\hat{\lambda}_{FIX}$ is zero, would be impractically large (> 8,000). A practically relevant comparison of performance should therefore recognize, as done here, that the estimated bias in $\hat{\lambda}_{FIX}$ in simulated sampling with a relative small number of replications may not be zero.

Root mean squared errors

The performance with respect to RMSE and $n = 15$ indicated a consistent ranking of the four estimators across the 58 patterns (Spearman rank cor-

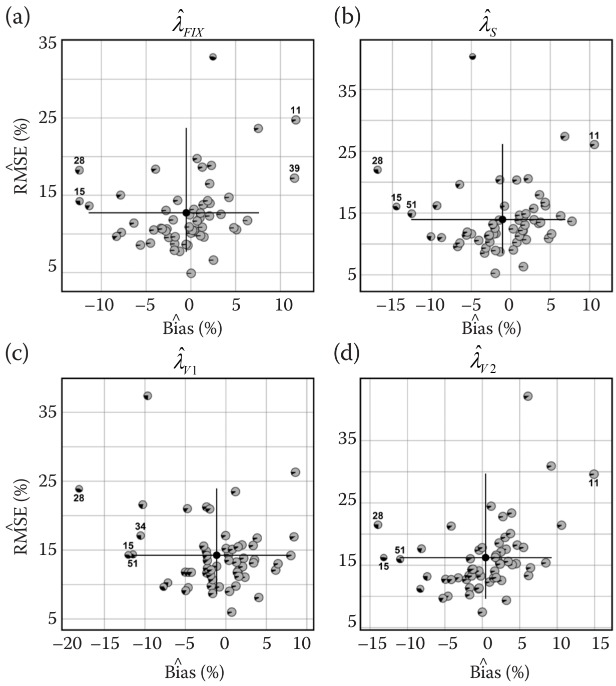

Fig. 5. Bubblechart summaries of estimates of BIAS, RMSE, and cCI95 with $\hat{\lambda}_{FIX}$(a), $\hat{\lambda}_{S}$(b), $\hat{\lambda}_{V1}$(c) and $\hat{\lambda}_{V2}$(d) estimates (grey-tone area of a pie chart indicates cCI95 with the black sliver indicating 1-cCI95, overall (sites) means of the estimates of BIAS and RMSE are indicated by a black dot, horizontal and vertical lines cover an interval from the 0.05 to the 0.95 quantile of the site-specific estimates of BIAS and RMSE)

relations of 0.97–0.98). This is also apparent in Fig. 5, where the four estimator specific-scatter plots show a great degree of resemblance. The overall average estimate of relative RMSE was 13% for $\hat{\lambda}_{FIX}$14% for $\hat{\lambda}_{S}$ and $\hat{\lambda}_{V1}$ and 16% for $\hat{\lambda}_{V2}$. In 54 patterns the lowest RMSE was obtained with $\hat{\lambda}_{FIX}$ and the largest was obtained with $\hat{\lambda}_{V2}$. In 39 cases $\hat{\lambda}_{S}$ ranked the second and $\hat{\lambda}_{V1}$ the third; in 15 cases their ranking was reversed. As for bias, the site-specific estimates of relative RMSE varied considerably; from 5 to 33% with $\hat{\lambda}_{FIX}$. Wider and right-shifted ranges were obtained with $\hat{\lambda}_{S}$ (5–40%), $\hat{\lambda}_{V1}$ (6–37%), and $\hat{\lambda}_{V2}$(8–42%). The interquartile range of site-specific estimates of relative RMSE was 4% for $\hat{\lambda}_{FIX}$ and 5% for the FCS estimators.

With the exception of $\hat{\lambda}_{V1}$ increasing the sample size from 15 to 30 achieved approximately (±2%) the expected (average) reduction in RMSE of $(1-2^{-0.5}) \times 100\% \cong 29\%$ for an unbiased estimator. The reduction for $\hat{\lambda}_{V1}$ was only 21%. The reduction in RMSE was equally reflected in the range of estimates. Results in terms of rankings were not materially affected by the increase in sample size.

Lowering the sample size from 15 to 9 increased, as expected, the RMSE: least (15%) in $\hat{\lambda}_{FIX}$, and most (30%) in the FCS estimators. The larger increase in FCS estimators reflects a combination of a small increase in bias and a poorer model fit. Apart from the increase in RMSE, all trends across sites and ranking of estimators were very similar to results with $n = 15$.

The relative frequency with which an estimate of RMSE for $\hat{\lambda}_{S}$ was less than the RMSE estimated for $\hat{\lambda}_{FIX}$ was 0.44 in this study. Corresponding results for $\hat{\lambda}_{V1}$ and $\hat{\lambda}_{V2}$ were 0.43 and 0.42.

Empirical and analytical estimates of variance

A linear regression, with the empirical variance in 1,200 replications of $\hat{\lambda}_{S}$ as the dependent variable ($n = 15$), and the average analytical site-specific estimate of variance as the explanatory variable, achieved an adjusted R-squared value of 0.91, a slope of 1.01 (± 0.04), and a non-significant ($P = 0.53$) intercept of 0.00. Comparable results for $\hat{\lambda}_{FIX}$ were: a slope of 0.97 (± 0.004); a non-significant ($P = 0.83$) intercept of 0.00; and an adjusted R-squared value of 0.99. Regression models for $\hat{\lambda}_{V1}$ and $\hat{\lambda}_{V2}$ had estimated slopes of 1.18 (± 0.01) and 1.19 (± 0.03), both significantly ($P < 0.002$) greater than 1.0. A graphical display of these results is in Fig. 6.

The above regression results were virtually repeated with $n = 30$. However, with $n = 9$ the average of the analytical variance tracked the observed variance less well than with $n = 15$. The explained

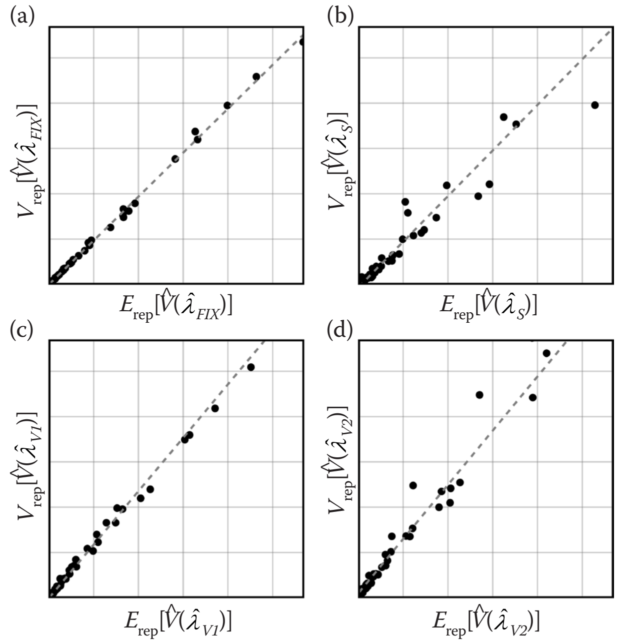

Fig. 6. Observed variance of $\hat{\lambda}$ viz. $V_{rep}(\hat{\lambda})$ plotted against the expected value of the analytical estimator of variance ($E_{rep}[\hat{V}(\hat{\lambda})]$), dashed one-to-one line is added as a visual aide

variance dropped by approximately 3%, and estimates of slopes were either slightly (4%) lower ($\hat{\lambda}_S, \hat{\lambda}_{FIX}$) or slightly (5%) higher ($\hat{\lambda}_{V1}, \hat{\lambda}_{V2}$). As for n =15 and n = 30, no intercept was significantly different from zero.

Achieved coverage rate of 95% confidence intervals

Normal theory 95% confidence intervals computed on the basis of $\hat{\lambda}_S$ and $\hat{V}\left(\hat{\lambda}_S\right)$ achieved an average coverage of 0.93 (site range: 0.46–1.00). With fixed area sampling the average coverage matched the intended coverage to within 0.03% but site-specific results varied from 0.57 to 1.00. Computed CI95s from sampling with virtual fixed area plots were in most cases too short with an average coverage less than intended (88% for $\hat{\lambda}_{V1}$, and 90% for $\hat{\lambda}_{V2}$). Again, these averages cover a wide range (0.54–0.96) of site-specific results.

Increasing the sample size from 15 to 30 increased the average coverage of CI95s in all four density estimators by approximately 2%. More importantly the minimum site-specific coverage improved by 12%. When dropping the sample size from 15 to 9, the achieved coverage decreased by approximately 4% in all four density estimators. Otherwise trends and rankings across sites with n = 30 or n = 9 were similar to those with n =15.

DISCUSSION

To consider a distance from a randomly selected point to the j^{th} nearest trees as a survival time may have intuitive appeal but does not, in and by itself,

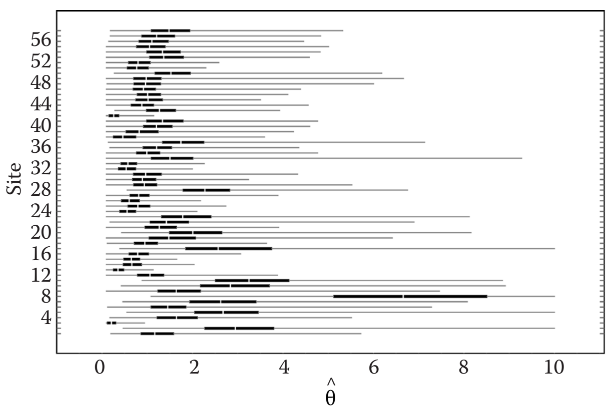

Fig. 7. Within-site distribution of the estimated variance $\hat{\theta}$ in shared random effect frailty. The sites are listed in order (1 to 58) of processing. The first four are simulated point patterns used for the model development

offer any advantages in the current context of constructing an FCS estimator of stem density. The key to the relative success of the new proposed estimator rests with the joint estimation of the distribution of the j = 1, …, k distances as a function of log(j) and inclusion of a random location effect (in density) called a shared frailty. A random location effect cannot be estimated from just the distances to the k^{th} nearest tree. Replications are needed, and here they are in the form of all k distances. There are two disadvantages: (i) the location effect is estimated for an ensemble of ordered distances and not specifically for the distance to the k^{th} nearest tree; and (ii) the estimated parametric distribution function of distances is conditional on the order (j) of the distance, hence estimates of means and variances require a calibration to the distribution of the kth nearest trees.

Adopting a survival function modelling framework for an FCS estimator of stem density offers an analyst a wide spectrum of flexible survival functions and options to exploit auxiliary variables (Gutierrez 2002; Li, Ryan 2002, 2004; Banerjee et al. 2003; Wienke 2010). It is known that distinct spatial processes generate distinct distributions of distances to the nearest l^{th} element (l = 1, …, k) (Illian et al. 2008; Oedekoven et al. 2014). For a first-order stationary spatial process generating a point pattern, the ensuing point density is closely linked to the distribution of distances from a randomly selected location to its k nearest neighbours. Identifying the link is a complex challenge, unless the point process is compatible with complete spatial randomness (Thompson 1956; Isham 2010). In this study, a functional link between the four parameters in the survival function, and the point density of a site was not apparent. Therefore the benefit of a survival function was limited to the estimation of the parameter in the assumed gamma distribution of the shared frailty viz. heterogeneity in local point density. A benefit that made the performance of $\hat{\lambda}_S$ run almost parallel to that of $\hat{\lambda}_{FIX}$. A consistent relative performance against a benchmark is important from a practical perspective. It allows an analyst a better informed decision when pros and cons of fixed area versus fixed count sampling are considered. A widely fluctuating performance of simpler FSC estimators (Moore 1954; Morisita 1954, 1957; Persson 1964; Eberhardt 1967; Pollard 1971; Cox 1976; Delince 1986; Kleinn, Vilčko 2006b) is an important deterrent except in a few cases when an analyst knows what to expect from a chosen FSC estimator (Lynch, Rusydi 1999; Kleinn et al. 2009).

The use of a survival function in an FCS estimator of stem density may offer some additional tangible advantages, including but not limited to: use of density-dependent explanatory variables (Nielson et al. 2004); ease of handling censored distances along borders of a finite area population (Lynch 2012); including conditional probabilities of observation (Barbour, Gerritsen 1996; Doberstein et al. 2000; Farnsworth et al. 2002).

The variance of the random and shared frailty effects (θ) was statistically significant in all individual estimates, suggesting either a significant over-dispersion in local point density (θ < 1) or a significant under-dispersion (θ > 1) relative to that of the complete random point pattern. As expected, $\hat{\theta}$ was related to the among-plot variance of $\hat{\lambda}_{FIX}$. For the 58 sites, a second-degree polynomial with $\hat{\lambda}_{FIX}(site)$ as the dependent and log($\hat{\theta}(site)$) as the explanatory variable achieved an R-squared value of 0.97 with all polynomial terms highly significantly different from 0 ($\hat{t} \geq 15.7$, $P < 0.00$). Larger values of $\hat{\theta}$ (*site*) were indicative of a larger within-site variation in local point density. Fig. 7 illustrates, for each site, the distribution of estimated values of θ. Sites with a quasi-regular point pattern are characterized by low values of θ with a mean in the interval [0.25; 0.5] and an interquartile range of approximately 0.2. Strongly clustered sites (e.g. 2, 8, 11, and 17) display not only a large average value of θ, but equally a large among-sample variation in estimates of θ. Presumably θ is also related to Ripley's *K*-function (Illian et al. 2008) and could therefore serve as quantitative indicator of the variance in local density.

Using a survival function facilitated the construction of a bias-correction term ($\tilde{c}$) intended to counteract the effect of using the predicted area of the smallest circle including, on average, the *k* nearest elements at a randomly chosen sample location (Pollard 1971; Cox 1976; Delince 1986; Picard et al. 2005; Kleinn, Vilčko 2006b). A risk-based approach to constructing a bias-correction term was adopted (Casella, Berger 2002): The risk of using the expected distance as the radius in an assumed fixed area circular plot for the calculation of an FCS density estimator was equated to the likelihood that the distance is less than expected, times the likelihood that it is greater. Attempts to replace this risk-based correction term with the more familiar 0.5 (Kleinn 1996) only resulted in an increase in estimates of bias.

From a modelling perspective, this study suggests that improvements in FCS estimators must come from not only in modelling the distribution of distances but also the within-site variance in local density. A direct modelling of the latter in the form of a marginal distribution of density has met with mixed results (Magnussen et al. 2008b; Magnussen 2012b). Estimating the distribution of frailty appears more promising. A side-effect of estimating a model for frailty and a model for distances is the increase in the number of parameters to be estimated. This puts upward pressure on the required minimum sample size for obtaining results with an acceptable precision. Results from this study suggest a minimum sample size of 12 for the proposed estimator.

The proposed FCS estimator of density appears to hold a small but important advantage over existing practical FCS alternatives, at least in terms of RMSE and cCI95, which are important to both an analyst as well as to users of FCS results. While the marginal advantages across 58 point patterns may seem slim, the correlation among results with $\hat{\lambda}_{FIX}$ and $\hat{\lambda}_S$ was considerably stronger than with the alternatives.

The reported performance of $\hat{\lambda}_S$ has been quantified in terms of averages across sites. However, a strong correlation among site-specific results with $\hat{\lambda}_{FIX}$ and $\hat{\lambda}_S$ supports the expectation of stability in relative performance. Nevertheless, the wide range in performances of the four estimators across sites is an area of concern. The poor performance of all four estimators on a handful of sites – with either a large amount of within-site variation in local point density or a patchy mosaic of areas with distinct differences in point density – reflects that a precise and an accurate estimate of density from such sites is only possible with large samples (e.g. $n > 100$).

The large variation in the 58 spatial point patterns used in this study vouches for a robust assessment of the proposed FCS estimator of density and allows a generalization of the results to practice, not only in forestry, but also to surveys in biology and ecology where FCS can be attractive (Doberstein et al. 2000; Picard et al. 2005; Steinke, Hennenberg 2006; White et al. 2008).

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