Harvest scheduling with spatial aggregation for two and three strip cut system under shelterwood management

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ABSTRACT: We propose a spatial aggregation method to solve an optimal harvest scheduling problem for strip shelterwood management. Strip shelterwood management involves either a two-cut system with a preparatory-removal cut cycle, or a three-cut system with a preparatory-establishment-removal cut cycle. In this study we consider these connected sequential cuts as one decision variable, then employ conventional adjacency constraints to seek the best combination of sequential cuts over space and time. Conventional adjacency constraints exclude any spatially-overlapped strips in the decision variables. Our results show the proposed approach can be used to analyze a strip shelterwood cutting system that requires "connectivity" of management units.

Keywords: aggregation; connectivity; GIS; optimization model; spatial forest planning; wind-thrown risk

Forest managers are increasingly confronted with complex and diverse management problems such as the loss of biodiversity, disruption of ecosystem services, and damage from natural disturbances. To mitigate the damage- or risk-associated with these management issues, it is often prudent to consider allocation of management activities over space and time because any management activity in a given management unit could impact other spatially-related units. For example, natural disturbances such as windthrow, fire, or insect infestation involve spatial dynamics that can spread a damage-causing factor over space and time. Thus, withholding corrective management action on one site could increase risk of loss on other sites.

Since the late 1980's, increasing emphasis on meeting ecological goals has pushed the development of optimal forest management plans that specify the location and timing of management activities. Many studies have formulated spatially-constrained harvest scheduling problems that search for spatial harvest patterns that prevent excessively large openings resulting from the harvest of adjacent forest stands. Various mathematical programming models for a spatially-constrained harvest scheduling problem have been developed. Early efforts include Sessions and Sessions (1988), Clements et al. (1990), Nelson and Brodie (1990), Yoshimoto et al. (1994), Murray and Church (1995).

This type of problem can be formulated and solved using exact solution techniques by employing an adjacency constraint structure. However, as the number of management units, planning periods, and exclusion periods increase, the number of such constraints also increases and the problem becomes too large to be solved by the exact solution techniques of integer programming. As a result, several methods to reduce redundant adjacency constraints have been proposed for solving adjacency constrained problems. For example, Yoshimoto and Brodie (1994) developed an algorithm

Supported by the Ministry of Education, Culture, Sports, Science, and Technology of Japan, Grant No. 18402003.

to solve this type of problem using an adjacency matrix. They reduced the number of adjacency constraints by using matrix algebra and taking advantage of the symmetric nature of the matrix. Early adjacency studies focus on dispersion of harvest units. If no large opening is created, fewer environmental impacts are assumed to result from harvest activities (Snyder, Revelle 1997). Dispersion of harvest units may well be dealt with by conventional adjacency constraints that prohibit harvesting any two adjacent units simultaneously. Dealing with current management issues, however, often requires explicit consideration of spatial patterns, such as "connectivity" of management units that results from certain vegetation relationships. For example, the connectivity of old growth forests must be maintained to protect corridors that constitute critical habitat for certain wildlife species. In such a case, it is important to consider not only directly adjacent units, but also indirectly adjacent units that may be integral to maintaining overall connectivity.

In this study, we propose a spatial aggregation method to solve an optimal harvest scheduling problem subject to "connectivity" requirements. We formulate our approach as a spatial forest management problem and apply it to strip shelterwood management, a forest management regime commonly used in Europe (Matthews 1989). The strip shelterwood management regime specifies the sequence of management activities, which generally progress in a sequential fashion into the prevailing wind. Most commonly applied shelterwood management regimes involve either a two-cut system with a preparatory-removal cut cycle, or a three-cut system with a preparatory-establishment-removal cut cycle, which progress from windward to leeward. Under the three-cut system, for example, the strip-by-strip cut cycle positions a "preparatory cut strip," "establishment cut strip," and "removal cut strip" over space and time. Therefore, the strips are lined-up from "preparatory cut strip" to "removal cut strip" in a specific directional order, which creates a spatial forest structure that protects against wind damage (Fujimori 2001). We utilize conventional adjacent constraints to formulate a strip aggregation optimization problem for strip shelterwood management.

General problem specification

We formulate a simple spatially constrained problem within a 0–1 integer programming framework without considering harvest flow. We assume the objective is to maximize the total cut volume from all forest stands over the planning period. Constraints include land accounting, as well as spatial restrictions to avoid harvesting two adjacent strips during the same period. Let $\boldsymbol{X} = (\boldsymbol{x}_1,....,\boldsymbol{x}_m)' = (\tilde{\boldsymbol{x}}_1,....,\tilde{\boldsymbol{x}}_n)$ be an $(m \times n)$ dichotomous decision matrix with m as the number of stands and n as the number of treatments for one stand, and ′ denotes the transpose, where x_i is the i-th row vector of $\tilde{\boldsymbol{x}}_i$ for the i-th stand and $\tilde{\boldsymbol{x}}_j$ is the j-th column vector for the j-th treatment. An element of X is thus defined by,

$$x_{i,j} = \begin{cases} 1 & \text{if the } j\text{-th treatment is implemented for the } i\text{-th stand} \\ 0 & \text{otherwise} \end{cases}$$

Although Model I formulation by Johnson and Stuart (1989) used a decision vector to meet the general formulation requirements of linear programming, we introduced a decision matrix to clearly assign the treatment to strips by the row and column of X. The objective here is given by,

$$Z = \max_{\boldsymbol{X}} \operatorname{tr}(\boldsymbol{C}'\boldsymbol{X}) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{i,j} \times x_{i,j},$$

where:

C – $(m \times n)$ coefficient matrix and its element,

$c_{i,j}$ – total volume obtained by the decision $x_{i,j}$.

Given a planning period of 10, with six periods as a minimum cutting cycle, Table 1 shows an example of 20 treatments for one stand. The treatment regime for one stand can be summarized as, "cut the fifth strip in period three."

To formulate land accounting constraints, which require at most one treatment for each stand, we have the following:

$$1_n' x_i \le 1, \quad i = 1, 2, ..., m,$$

where:

1_n – $(n \times 1)$ vector with a value of 1.

"No treatment" is also considered in the decision variable.

Adjacency constraints prevent two adjacent strips from being cut during the same period. Following Yoshimoto and Brodie (1994), we have:

$$M \times \tilde{x}_j \le m_0, \quad i = 1, 2, ..., m,$$

where:

$m_0 = A \times 1_m$,

$M = A + diag(m_0)$.

and an element of the above adjacent matrix (A) is defined by

$$\boldsymbol{a}_{i,j} = \begin{cases} 1 & \text{if } j \in NB_i \\ 0 & \text{if } j \notin NB_i \end{cases}$$

where:

NB_i – set of stands adjacent to the i-th stand.

Table 1. Example of treatments

	Treatment No.	Decision variable	Coefficient	Period									
				1	2	3	4	5	6	7	8	9	10
Treatment	1	$x_{i,1}$	$c_{i,1}$	X	0	0	0	0	0	0	0	0	0
	2	$x_{i,2}$	$c_{i,2}$	X	0	0	0	0	0	X	0	0	0
	3	$x_{i,3}$	$c_{i,3}$	X	0	0	0	0	0	0	X	0	0
	4	$x_{i,4}$	$c_{i,4}$	X	0	0	0	0	0	0	0	X	0
	5	$x_{i,5}$	$c_{i,5}$	X	0	0	0	0	0	0	0	0	X
	6	$x_{i,6}$	$c_{i,6}$	0	X	0	0	0	0	0	0	0	0
	7	$x_{i,7}$	$c_{i,7}$	0	X	0	0	0	0	0	X	0	0
	8	$x_{i,8}$	$c_{i,8}$	0	X	0	0	0	0	0	0	X	0
	9	$x_{i,9}$	$c_{i,9}$	0	X	0	0	0	0	0	0	0	X
	10	$x_{i,10}$	$c_{i,10}$	0	0	X	0	0	0	0	0	0	0
	11	$x_{i,11}$	$c_{i,11}$	0	0	X	0	0	0	0	0	X	0
	12	$x_{i,12}$	$c_{i,12}$	0	0	X	0	0	0	0	0	0	X
	13	$x_{i,13}$	$c_{i,13}$	0	0	0	X	0	0	0	0	0	0
	14	$x_{i,14}$	$c_{i,14}$	0	0	0	X	0	0	0	0	0	X
	15	$x_{i,15}$	$c_{i,15}$	0	0	0	0	X	0	0	0	0	0
	16	$x_{i,16}$	$c_{i,16}$	0	0	0	0	0	X	0	0	0	0
	17	$x_{i,17}$	$c_{i,17}$	0	0	0	0	0	0	X	0	0	0
	18	$x_{i,18}$	$c_{i,18}$	0	0	0	0	0	0	0	X	0	0
	19	$x_{i,19}$	$c_{i,19}$	0	0	0	0	0	0	0	0	X	0
	20	$x_{i,20}$	$c_{i,20}$	0	0	0	0	0	0	0	0	0	X

X – harvesting while 0 denotes no harvesting

Using the formulation above, we can allocate treatments over space without harvesting adjacent stands in the same period.

Demonstrative case study

We present an empirical example of the spatial arrangement of aggregated strips to mitigate wind damage risk. Our study site is part of a forest managed by the School Forest Enterprise at the Technical University in Zvolen, Slovakia. The site consists of six management units (MU) that are collectively 163.73 hectares (Fig. 1a). According to Slovak Forestry Act No. 326/2005, these units should be managed under a strip shelterwood silvicultural system that supports natural regeneration. Under the strip shelterwood system, MUs are first divided into a strip window where the unit is harvested over the regeneration period in a series of like-sized, uniformly staggered linear strips that advance progressively through units in one direction, most often into the prevailing wind. Strip width is generally set at four times the average dominant height of the target forest stand. For this site, a total of 58 strips were created (Fig. 1b). The average size of these strips was 2.82 ha (min 1.04 ha, max 5.53 ha). The strip shelterwood management regime involves a two cut system with a preparatory-removal cut cycle, or a three cut system with a preparatory-establishment-removal cut cycle. In either case, a cut cycle will progress from the windward to leeward direction.

The two-cut system begins with a preparatory cut for a windward strip. After a few years (e.g. five years), a removal cut will be conducted in this strip and a preparatory cut will simultaneously be implemented in the leeward adjacent strip. A few years later, when the removal cut for this leeward adjacent strip is completed, a continuous cut sequence (preparatory-removal) will be initiated, starting from the strip adjacent to the one where the removal cut is completed (Table 2). By conducting the preparatory cut and removal cut in two adjacent strips against the prevailing wind, this system creates a spatial forest structure that mitigates wind damage risk by gradually increasing average tree height from the windward to leeward direction. If the

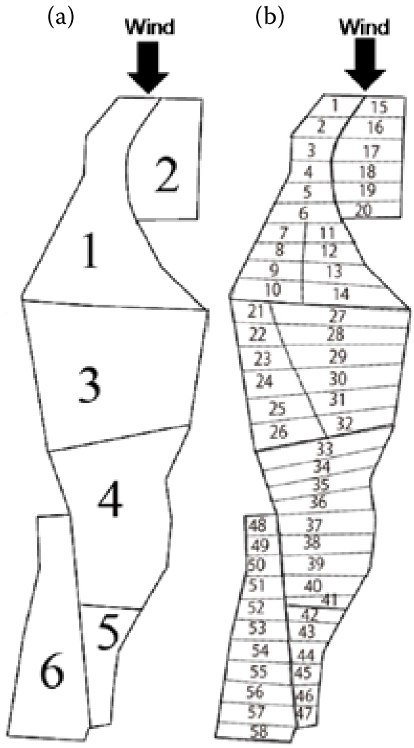

Fig. 1. The study area landscape with management units (MUs) (a), and with strips (b)

regeneration period in this example is three 10-year planning periods (30 years), the time spans between preparatory and removal cuts is five years. Then, two cuts are completed within 10 years and the removal cut is completed in five adjacent strips within the regeneration period of 30 years.

The three-cut shelterwood system consists of a preparatory, establishment, and removal cut. Like the two-cut system, three sequential cuts must be completed within 10 years (within a regeneration period of 30 years, the removal cut is completed on seven adjacent strips). Therefore, in this example, the time span between each cut is three to four years. As in the previous system, the sequence of three cuts (preparatory, establishment, and removal) starts from the windward strip (Table 3). With a time lag of three to four years, the sequence of three cuts is initiated on leeward adjacent strips. A few years later, another sequence of three cuts will be initiated on further leeward adjacent strips. At the end of the first period – for a given set of three adjacent strips – the removal cut is completed on the most windward strip, the establishment cut on the middle, and the preparation cut on the leeward strip. Therefore, this system also creates a height-sorted spatial structure by assigning the cut sequence in each strip with a time lag.

The management goal of both systems is to maintain a spatial forest structure that protects stands from wind damage while maximizing timber harvest. This shelterwood management problem can be categorized as a spatially constrained harvest scheduling problem, where a sequential cut over space and time in adjacent strips is considered one decision variable. Generally, for a given unit (the focal unit), unit aggregation begins by connecting each adjacent unit based on the wind direction. Then, strips are aggregated from a windward to leeward direction with the most upwind strip set as the focal strip. Thus, adjacency relationships among strips are unidirectional (Fig. 2).

Mathematical programming formulation

In order to secure sequential cuts on adjacent strips for risk mitigation during the regeneration period, we aggregate five strips in one unit for the two-cut system, and seven for the three-cut system. Then, we apply adjacency constraints to prevent any two overlapped aggregated units from being selected at the same time. Basically, this aggregation requires "connectivity" of strips. For example,

Table 2. Example of allocation and cutting progress of two-cut shelterwood system

Period	Wind ⇒													
	cut	strip												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	P					P							
	2	R	P				R	P				P		
2	3		R	R				R	P			R	P	
	4			R	P				R	P			R	P
3	5				R	P				R	P			R
	6					R					R			

R – removal cut; P – preparatory cut

Table 3. Example of allocation and cutting progress of three-cut shelterwood system

Period	Wind ⇒													
	cut	strip												
		1	2	3	4	5	6	7	8	9	10	11	12	13
	1	P												
1	2	E	P						P					
	3	R	E	P					E	P				
	4		R	E	P				R	E	P			
2	5			R	E	P				R	E	P		
	6				R	E	P				R	E	P	
	7					R	E	P				R	E	P
3	8						R	E					R	E
	9							R						R

R – removal cut; P – preparatory cut

in the case of the two-cut system, forest managers must complete management activities for five connected strips together. We additionally consider constraints that prohibit cutting two adjacent focal strips at the same time. Then, we search for an optimal aggregation pattern that maximizes the number of strips treated (minimizing the number of strips left un-aggregated and un-managed), subject to spatial constraints. Given the management objective described above, we formulate our strip shelterwood scheduling problem using a 0–1 integer programming framework as follows:

Let a candidate of aggregated unit AU_j be a set of connected strips when aggregation starts from any strip as a focal strip toward a leeward direction. Let us also define $NB(i)$ as the index number of a strip adjacent to the i-th strip against the prevailing wind. Then, after completing the recursive operation four times – for the two-cut system – we have the following set consisting of five strips:

$$AU_j = \{i, NB(i), NB(NB(i)), NB(NB(NB(i))), NB(N(NB(NB(i))))\}.$$

For the 1st, 2nd, and the 3rd strip in Fig. 1b – for the two-cut system – we have the following:

$AU_1 = \{1, 2, 3, 4, 5\}$, $AU_2 = \{2, 3, 4, 5, 6\}$,
$AU_3 = \{3, 4, 5, 6,7\}$, $AU_4 = \{3, 4, 5, 6,11\}$.

There are a total number of 66 aggregated units because strips 6 and 10 are branched – they are connected to more than one strip (strip 6 is connected to both strips 7 and 11, while strip 10 is connected to strips 21 and 27; refer to Fig. 1b). As a result of this branching, the number of decision variables is greater than the total number of strips in the unit. Note that the subscript for the aggregated unit is conveniently specified so as to identify all candidates. Likewise, for the three-cut system, after completing the recursive operation six times, we have a set of seven strips:

$$AU_j = \{i, NB(i), NB(NB(i)), ... NB(NB(NB(NB(NB(NB(i))))))\}.$$

For the 1st strip in Fig. 1b – for the three-cut system – we have the following:

$AU_1 = \{1, 2, 3, 4, 5, 6,7\}$, $AU_2 = \{1, 2, 3, 4, 5, 6,11\}$.

The total number of the aggregated units is 70. When we develop aggregated units for all strips, some units overlap with others (as in Fig. 3). In other words, a strip that is a member of the i-th aggregated

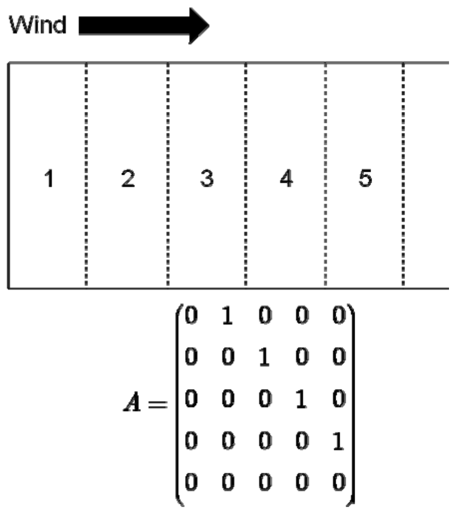

Fig. 2. Adjacent structure

unit, AU_i, will also be a member of another aggregated unit. These aggregated units cannot be chosen simultaneously; therefore, in this study we exclude overlapping units by applying conventional adjacency constraints with the following adjacency matrix: $A^* = \{a^*_{i,j}\}$,

where:

$$a^*_{i,j} = \begin{cases} 1 & \text{if } AU_i \cap AU_j \neq \varnothing \\ 0 & \text{if } AU_i \cap AU_j = \varnothing \end{cases}$$

Let us introduce the decision variable y_j, for the j-th aggregated unit.

$$y_j = \begin{cases} 1 & \text{if the } j\text{-th aggregated units is selected} \\ 0 & \text{otherwise} \end{cases}$$

Then, assume that our objective is to maximize the number of strips treated over the regeneration period.

$$Z = \max_y \sum_{j=1}^{N} w_j y_j,$$

where:

w_j – number of strips in AU_j,

N – total number of the aggregated units.

By introducing the above objective function and applying adjacency constraints, we can solve the strip shelterwood management problem.

$$M^* \times y_j \leq m_0, \qquad i = 1, 2, \ldots, m,$$

$$m_0 = A^* \times 1_N,$$

$$M^* = A^* + \text{diag}(m_0).$$

We use CPLEX (Ilogs 2003) to search for an optimal aggregation pattern. Fig. 4a shows the optimal solution that specifies the optimal spatial pattern of the two-cut system. Following the optimal aggregation pattern, 11 aggregated units were selected for management and four strips were left un-aggregated and un-managed. Each aggregated unit consists of five adjacent strips except unit 52, which contains four strips located at the lower end of the study site.

Fig. 4b shows the optimal aggregation pattern for the three-cut system. Seven aggregated units were selected for management and 11 strips were left un-aggregated and un-managed. Each aggregated unit consists of seven adjacent strips except unit 28, which contains 5 strips located at the upper end of the study site.

Our results show that for both the two-cut and three-cut systems, an aggregated unit with fewer strips is also selected in the optimal aggregation pattern. This is because our model considers unidirectional adjacency that limits the possible aggregation patterns on the margins of the study site, but results in greater profit.

Comparing the two systems shows that the tighter constraints necessary for aggregating seven strips (as compared to five) results in more un-managed strips. Therefore, it is possible that less timber volume will be removed under the three-cut system. Our experimental study demonstrates that the proposed aggregation approach is a valid means of solving spatial management optimization problems designed to mitigate windstorm risk.

Concluding remarks

In this study we proposed a new spatial aggregation method to solve an optimal harvest scheduling problem for strip shelterwood management intended to mitigate windstorm risk. The proposed

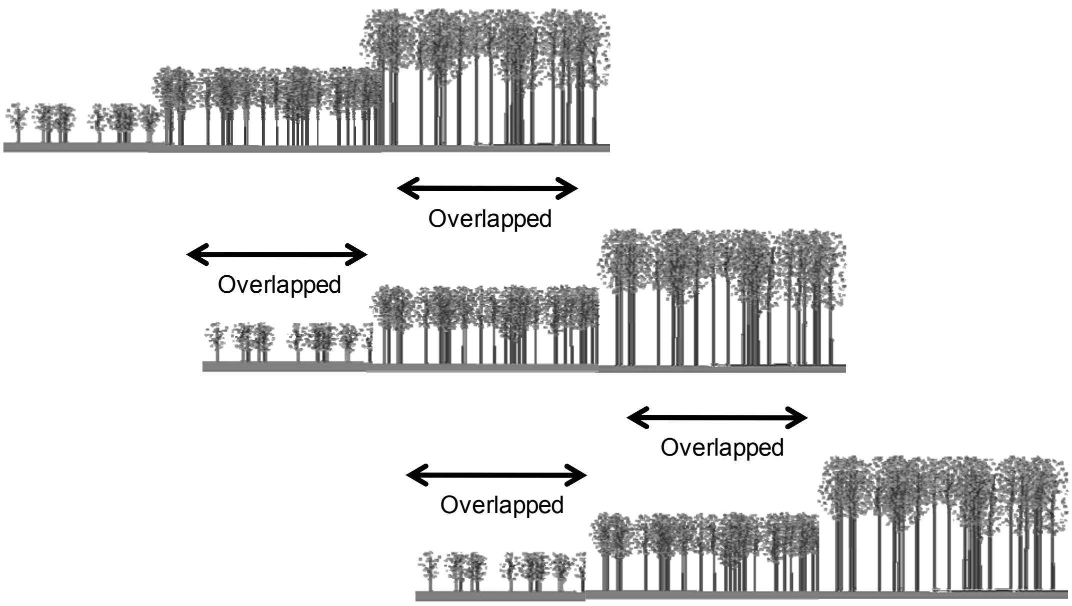

Fig. 3. Overlapped strips (figure was created using the programs Suppose – Crookston N.L. and SVS – McGaughey R.J.)

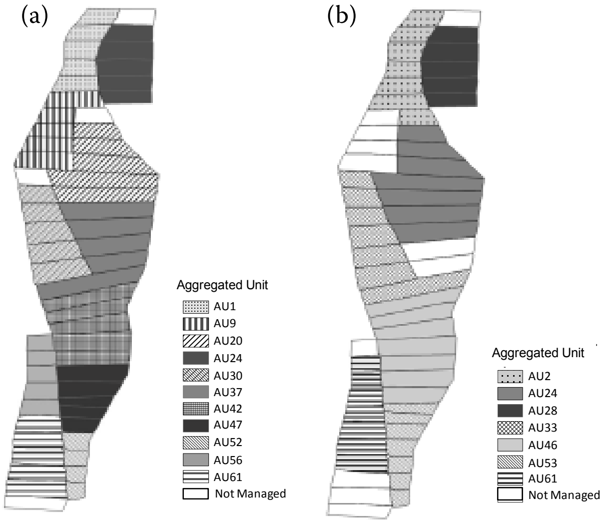

Fig. 4. Optimal aggregation pattern of two-cut system (a), and three-cut system (b)

method utilizes sequential strip aggregation for each strip, and treats its aggregated unit as one decision variable for optimization. As a result, the number of decision variables becomes the same as, or more than, the number of strips, depending upon how many branches (i.e. aggregation patterns) exist from one strip. In our case study, there were two strips with two branched strips (strip 6 was connected to both strips 7 and 11, while strip 10 was connected to strips 21 and 27; refer to Fig. 1b). Thus the total number of decision variables (66 for the two-cut system and 70 for the three-cut system) was greater than the total number of 58 strips. In the final solution we applied ordinary adjacency constraints to avoid sharing strips among aggregated units.

We demonstrated our approach using a case study from a forest managed by the School Forest Enterprise at the Technical University in Zvolen, Slovakia. To reduce the risk of windthrow, adjacent strips were aggregated unidirectionally in a windward to leeward direction. Thus, strips were considered for adjacency only if they were adjacent to the leeward side of the previous strip. This is a special case of an adjacent structure commonly used (such as "Moore neighborhood adjacency") where strips- or units-sharing either lines or corners in any direction are considered to be adjacent (Childress et al. 1996).

Dealing not only with stand adjacency, but also with connectivity – or higher order adjacency – has been a complex problem for forest managers. Though many simulation approaches have been introduced for such complex problems, an optimization framework has not been proposed. Our approach would help formulate this complex spatial forest management problem within the framework of a conventional spatially constrained optimization model, and solve it using integer programming with an exact solution method.

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Received for publication May 19, 2010
Accepted after corrections March 21, 2011

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