Diameter structure of the stands of poplar clones

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ABSTRACT: The construction of a continuous mathematical model of frequency distributions of the diameters of trees of poplar clones Robusta and I-214 in dependence on tree diameter and mean diameter of stand is presented. Empirical material consists of diameter measurements on research plots from poplar regions in Slovakia. There were 90 plots for I-214 clone and 142 plots for Robusta clone. There were about 10–250 trees with mean diameter 2–70 cm on the research plots. The model was derived according to the three-parameter Weibull function. Its parameters were estimated by maximum likelihood method of the logarithm of the probability density function. Smoothed sample probability densities were processed in continuous mathematical models where the probability density of trees in stands is a function of their diameters and mean diameter of the stand. The method of regression smoothing of the parameters of Weibull function from sample sets in dependence on their mean diameter was used. In the whole range of mean diameters both clones have slightly left-skewed distribution with a relatively small variation range.

Keywords: diameter distribution function; frequency distributions of diameters; poplar clones; Robusta; I-214; Weibull function

Diameters and heights of trees, and mainly their structure characterize any forest stand very well. Particularly diameter structure is a very significant stand characteristic on the basis of which we can evaluate the stability, growth and volume production of stand as well as the structure of assortments, maturity, or it is possible to evaluate technologies of logging and timber transport. Diameter structure may be expressed very well by frequency distributions of tree diameters that quantify their distribution in diameter classes. In statistical terminology they are functions of probability density of tree diameters in stand. In the case of forest stands they are mostly unimodal, approximately bell-shaped curves. More frequently they are left-skewed asymmetrical and more platycurtic than leptocurtic. Their concrete shape is determined especially by tree species, age, and way of establishment or recent way of stand tending.

In recent forest practice several models of probability density of tree diameters in stand have been verified. In the last decades mainly distributions by Gamma, Beta and Weibull function were applied. Gerold et al. (1982) used the Gamma function for smoothing of diameter distribution of beech stands and its parameter *B* was correlated also in dependence on age and mean diameter. The introduction of maximum diameter into it has not increased significantly the accuracy of the relation. Zöhrer (1969) described in detail the Beta function and analyzed its characteristics as well as applied it to a concrete case. Kennel (1972) also used this function for beech stands. Čermák and Petráš (1984) used it for larch stands and correlated its parameters in dependence on mean diameter and diameter dispersion. Kupka (1985) considered the Beta function more suitable than Gamma function or Gauss function of normal distribution. Hui and Gadow (1996) applied a rare logistic differential three-parameter equation. Probably most experience is with the applications of Weibull function. Bailey and Dell (1973) used its two-parameter

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shape for smoothing of diameter distribution for 4 stands. HAFLEY and SCHREUDER (1977) stated that in addition to the Weibull function also Beta function or Johnson SB function are very flexible. Similarly GADOW (1984) examined the suitability of several functions. Based on fitting 774 measurements from 448 stands of the pine *Pinus patula* he reached the best results for the Weibull function followed by Johnson SB and Beta function. KUPKA (1987) also used the Weibull function in fitting the stands of spruce and pine. VAN LAAR and MOSANDL (1989) proved the advantages of Weibull function when compared with Beta function on young stands of oak with mean diameter 1–11 cm. Two- or three-parameter Weibull functions are considered equal. NAGEL and BIGING (1995) applied the two-parameter Weibull function for vast empirical material from 2,242 measurements of stands of oak, beech, spruce, Douglas fir and pine. They fitted its B and C parameters in dependence on mean and maximum diameter. HESSENMÖLLER and GADOW (2001) applied the Weibull function very well also for bimodal distribution of diameter frequencies of beech stands. Also KARCZMARSKI (2005) considered the Weibull function very suitable for fitting spruce primeval forests. SEDMÁK (2005) applied it in beech stands of the 3rd up to the 8th age classes in Slovakia, KANGAS and MALTAMO (2000) in pine stands of the 4th up to the 8th age classes in Finland. KÄRKI et al. (2000) expressed its two parameters in dependence on mean diameter d_g.

Recently MERGANIČ and STERBA (2006) applied the Weibull function with the method of moments estimation. PALAHI et al. (2006) used the two-parameter Weibull function for processing measurements from national forest inventory in Spain. NANOS and MONTERO (2002) compared Weibull function and Chaudhry-Ahmad function on the same empirical material. According to them the Weibull function gives relatively smaller systematic deviations. GAFFREY et al. (1998) processed the measurements of national forest inventory in Germany by means of Weibull and Gauss normal distribution function. In their opinion the Weibull function is more suitable for spruce and its parameters may be expressed in dependence on mean, minimal and maximal tree diameters. It follows from the above-mentioned overview that the Weibull function is applied very frequently and that it gives very satisfactory results.

The aim of our paper is to derive by means of Weibull function a continuous mathematical model of frequency distributions of tree diameters in poplar stands of the clones Robusta and I-214.

MATERIAL AND METHODS

Empirical material comes from poplar regions of Slovakia and comprises measurements of tree diameters $d_{1.3}$ on research plots of the poplar clones Robusta and I-214. Measurements come from various sources and most measurements are from the database of repeated measurements on research plots for the construction of yield tables (PETRÁŠ, MECKO 2001). Diameters of trees are arranged into 1 cm or 2 cm diameter classes. Others are from single measurements on research plots for assortment (PETRÁŠ et al. 2007). For Robusta in total 142 measurements were used, of them 111 are from the plots for the construction of yield tables. For I-214 there were 90 measurements in total, of them 66 on the plots for the construction of yield tables. In individual measurements there were about 10–250 trees on research plots with mean diameter d_g of about 2–70 cm.

For the processing of diameter structure possibilities of modelling by means of frequency distributions of diameters were used. With regard to recently published knowledge the three-parameter Weibull function was used. Its distribution shape is as follows:

$$F(d) = 1 - \exp\left(-\left(\frac{d-A}{B}\right)^{C}\right)$$

$$d > 0, A \le d < \infty, B > 0, C > 0 \qquad (1)$$

The first derivative of the distribution function is the function of probability density:

$$f(d) = \frac{C}{d} \times \left(\frac{d-A}{B}\right)^{C-1} \times \exp\left(-\left(\frac{d-A}{B}\right)^{C}\right) \qquad (2)$$

Expected probability n_i in i-th diameter classes was calculated from the distribution function as a difference of its values in neighbouring diameter classes:

$$n_i(d_i, \Delta d, N) = N \times \left[\exp\left(-\left(\frac{d-A-\Delta d}{B}\right)\right)^{C} - \right.$$
$$\left. - \exp\left(-\left(\frac{d-A-\Delta d}{B}\right)\right)^{C}\right] \qquad (3)$$

where:
d – tree diameter or mean of diameter class,
A, B, C – parameters of the function,
Δd – half width of diameter classes,
N – cumulative probability.

Parameter A gives position or more precisely determines minimal diameter and the beginning of distribution. If $A = 0$, then the two-parameter Weibull function is obtained with its beginning at point (0.0). Though parameter B gives the scale and parameter C the shape of the function, the final shape of diameter

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Table 1. Values of regression coefficients *a, b, c* of parameters *A, B, C* for the Robusta and I-214 clone

Parameter	Regression coefficient *a*	*b*	*c*	Coefficient of determination
Robusta				
A	0.0	0.5543553702	0.001536892608	0.812
B	0.0	0.4805956252	–0.001633090041	0.572
C	2.364030552	0.01863320808	–0.0002961459995	0.014
I-214				
A	–2.966082587	0.9892389267	–0.006063822563	0.704
B	3.234080557	0.005906625501	0.006686368106	0.650
C	3.059992375	–0.04188696209	0.0006136947451	0.075

distribution, it means its excess and asymmetry is determined by the combination of parameters *B* and *C* (Gadow 1984).

For each measurement a statistical model of diameter distribution was derived according to function (2). Parameters *A, B, C* of likelihood model *L* were calculated by maximum likelihood estimate according to the logarithm of probability density function *f*:

$$L(A, B, C, d) = \sum_{i=1}^{n} \ln f(A, B, C, d_i) \qquad (4)$$

The statistical package of QC-Expert programmes (Kupka 2004) was used. Likelihood of smoothing, it means the rate of accordance of empirical and model distribution of diameters was evaluated by P-P graph (Meloun, Militký 2002). In the case of full accordance of empirical distribution with the theoretical Weibull distribution the P-P graph is linear with slope 1.0 and zero section on the vertical axis. For lower accordance the empirical distribution oscillates around the theoretical one. Correlation coefficient expresses the closeness of their relation.

Empirical probability densities estimated according to function (2) were processed into continuous mathematical models where the probability density of trees in the stands n_i is the function of their diameters d_i and mean diameter of stand d_g according to the relation:

$$n_i = f(d_i, d_g) \qquad (5)$$

The method of regression smoothing of parameters *A, B, C* of the Weibull function from sample sets in dependence on their mean diameter d_g was used:

$$A, B, C = f(d_g) \qquad (6)$$

RESULTS AND DISCUSSION

Model and its shape

For all 232 sample sets of both clones, i.e. 142 measurements of Robusta clone and 90 measurements of I-214 clone, statistical models of the distribution of tree diameters with parameters *A, B, C* were derived according to function (2). The accordance of empirical distribution with the calculated theoretical Weibull distribution was evaluated by the correlation coefficient of probability P-P graph. The variability of calculated parameters *A, B, C* was analyzed in detail. Only their dependence on mean diameter of stand d_g according to relation (6) showed as significant. Regression dependence according to the polynomial of the 2nd degree was calculated separately for each parameter:

$$A, B, C\ (d_g, a, b, c) = a + b \times d_g + c \times d_g^2 \qquad (7)$$

with the following values of regression coefficients *a, b, c* (Table 1).

We may state with regard to the coefficient of determination that the closest dependence was found for parameter *A* and *B*. Dependence of parameter *C* on mean diameter is low. Their shapes are illustrated in dependence on mean diameter d_g in Figs. 1 and 2. Parameter *A* specifies the limitation of frequency distributions from the left; it means minimal diameters of trees. Their values change with higher mean diameter. For mean diameters 7–50 cm Robusta has lower initial diameters of trees than I-214 only by 1–3 cm. Parameters *B* and *C* intersect between the clones as well. In the middle part of mean diameters Robusta has slightly higher values. The final shape of frequency distributions is determined by the combination of both parameters. In Fig. 3 we can see

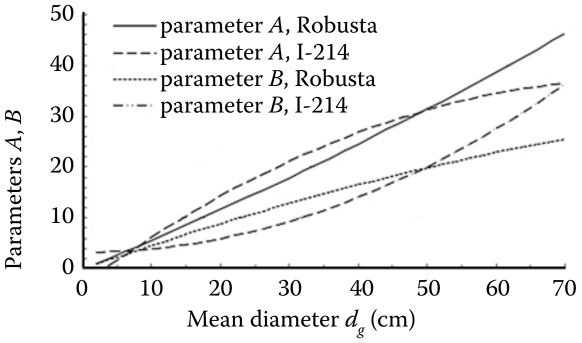

Fig. 1. Model values of the parameters A, B in dependence on stand mean diameter d_g of clones Robusta and I-214

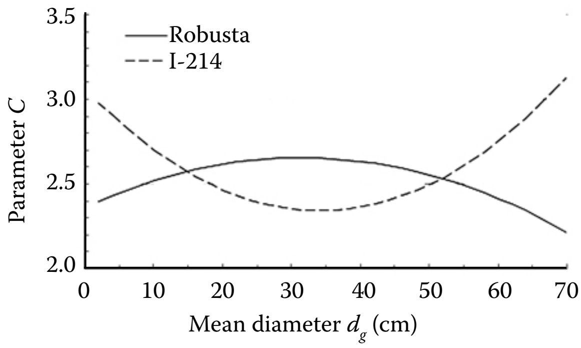

Fig. 2. Model values of the parameter C in dependence on stand mean diameter d_g of clones Robusta and I-214

that both clones have slightly left-skewed distribution with a relatively small variation range for mean diameters 20, 40 and 60 cm. I-214 has only slightly smaller ranges for mean diameter 20 and 40 cm, and Robusta for mean diameter 60 cm. A very small range of the diameters of both clones is confirmed also by their comparison with larch in Fig. 3. Larch frequency distributions were derived according to the Beta function by Čermák and Petráš (1984). They are differentiated for 3 classes of diameter range but in Fig. 3 they are given only for the lowest range. Larch as a light-demanding tree species has a very small range of tree diameters in comparison with the other tree species but poplar clones have even smaller ranges. Relatively to the values of mean diameters 20, 40 and 60 cm it is 145–117% for larch, 115–97% for Robusta and 85%–102% for I-214.

Model accuracy

In individual fitting of frequency distributions the accordance of empirical and theoretical distribution was evaluated according to the correlation coefficient from the graph P-P straight line. It reached the values within 0.92–1.0 almost in each fitting. It means that very high accordance was reached between the empirical and theoretical Weibull distribution. With regard to the fact that the continuous mathematical model according to relation (5) was derived by regression smoothing of model parameters A, B, C, its final accuracy was assessed according to the mean square error:

$$s_{ni} = \sqrt{\frac{\Sigma\,(n_i - \hat{n}_i)}{n - m}} \qquad (8)$$

where:

s_{ni} – the mean square error of probability density of the distribution of tree diameters in the stand,
n_i – empirical probability of the i-th diameter of trees,
$\hat{n}_i$ – model probability of the i-th diameter of trees,
n – the number of classes in diameter distribution,
m – the number of model parameters.

As Figs. 4 and 5 show, the greatest range of mean errors with the values of about 0.03–0.25 was found out for sets with the number of trees less than 30.

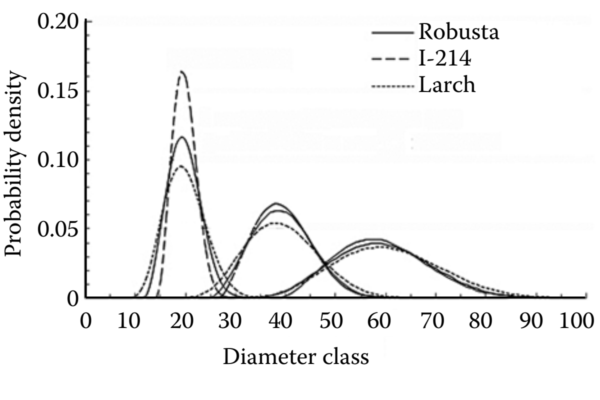

Fig. 3. Frequency distributions of tree diameters for stands of clone Robusta, I-214 and larch (Čermák, Petráš 1984) for mean diameters 20, 40 and 60 cm

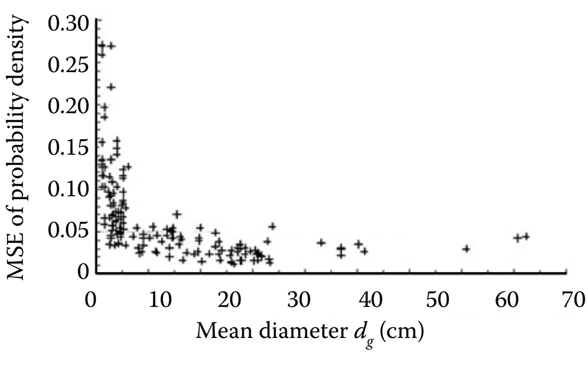

Fig. 4. Standard error of probability density of Weibull model in dependence on number of trees for clone Robusta

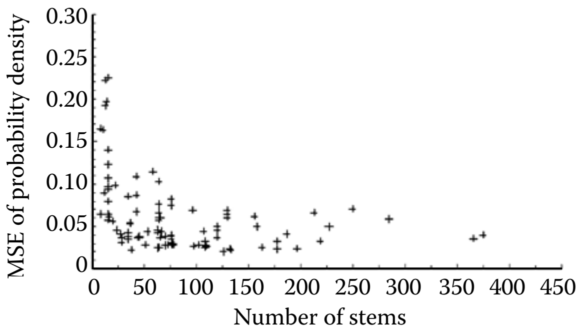

Fig. 5. Standard error of probability density of Weibull model in dependence on number of trees for clone I-214

For the number of trees 100 and more the errors are markedly smaller. Robusta has errors mostly within 0.01–0.05 and I-214 higher only by 0.02. The dependence of mean square errors on some other stand characteristics, e.g. on the mean diameter of stand, was not confirmed. If we express these absolute errors in % in relation to mean diameter d_g, we can state that the error is 0.1–0.5% for d_g = 10 cm and only 0.017–0.083% for d_g = 60 cm. Čermák and Petráš (1984) reported a similar range of errors for larch.

Though we did not compare the Weibull distribution function with other functions in our paper, we can say that the final model is smoothing distributions of tree diameters on research plots relatively well. Errors with higher values also occurred but they were found only on the research plots with a small number of trees. But it is important that these errors were not systematic, they were only random, and they were caused by a high variability of the number of trees in respective diameter classes on the plots.

We can also see in Fig. 4 that our models have a relatively large variation range of diameters despite the fact that the poplar clones have very simple genetics. The whole population of one clone is a vegetative progeny of only one and the same tree. Therefore we expected that their diameter variability would be smaller than for the stands of tree species with generative reproduction. But our model proves that the reason for diameter variability of trees in forest stands is not their genetic predispositions but concrete growth conditions, it means the location of each tree in the stand. The concrete location of a tree determines the quality of soil substrate including water capacity as well as determines the aboveground growth space that provides the necessary solar radiation. In comparison with larch trees (Fig. 3) or with some other tree species we can say that they have a larger variation range of diameters than poplar clones. The reason may be general growth capabilities of clones as well as higher heterogeneity of natural conditions under which they are growing. Poplar clones have relatively homogenous natural conditions in comparison with the other tree species mentioned above. They grow at low altitudes and mostly on the alluvia of rivers with relatively homogeneous soil environment.

CONCLUSIONS

The construction of a continuous mathematical model of frequency distributions of tree diameters in the stands of poplar clones Robusta and I-214 is presented. After very good experience of many authors we have chosen the three-parameter Weibull function for this purpose. Empirical material comes from poplar regions of Slovakia and comprises measurements of tree diameters on research plots arranged into 1 cm or 2 cm diameter classes. There were 90 measurements for I-214 and 142 measurements for Robusta. For the respective measurements there were about 10–250 trees on research plots with mean diameter d_g of about 2–70 cm.

For each measurement a statistical model of diameter distribution was derived according to the Weibull function. Parameters *A, B, C* of the likelihood model *L* were estimated according to relation (4) by maximum likelihood estimate of the logarithm of probability density function *f*. Estimated empirical probability densities were processed to continuous mathematical models where the probability density of trees in stands n_i is the function of their diameters d_i and mean diameter of stand d_g according to relation (5). The method of regression smoothing of parameters *A, B, C* of the Weibull function from empirical sets in dependence on their mean diameter d_g was used. For each parameter regression dependence according to equation (7) was derived. Both clones have slightly left-skewed distribution with a relatively small variation range in the whole range of mean diameters. This range is 97%–115% for Robusta and 85%–102% for I-214 relatively with regard to the values of mean diameter 20–60 cm.

The accordance between the empirical and theoretical Weibull distribution was evaluated according to the mean square error of probability density of diameters of trees in the stand of final model. The greatest range of mean errors with the values of about 0.03–0.25 was found for sets with the number of trees less than 30. With the number of trees 100 and more the errors are markedly lower. Robusta has errors mostly within 0.01–0.05 and I-214 higher only by 0.02. The dependence of mean square errors on some other stand characteristics, for example on the mean diameter of stand, was not confirmed.

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