Stand growth model using volume increment/basal area ratios

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ABSTRACT: Estimation of stand growth is crucial for forest planning. Estimations were usually done using fixed values, and recently growth equations have been used. An alternative is through stand growth models. The objective was to develop a simple model for *Nothofagus pumilio* stands with full density along site quality and age gradients. The sample was obtained from 125 stands. Data on forest structure and samples for tree-ring measurement were taken in all trees to estimate growths using biometric models previously developed. The growth values of each plot during the last twenty years were calculated to fit the model, using the ratio of total volume increment/basal area as an independent variable. The developed model gives a ratio between stand volume increment and basal area (m/year) in relation to the site quality and stand age. The statistics (r^2 = 0.819, mean error = 0.019, absolute mean error = 0.033), residual analysis and biological performance were satisfactory. The obtained stand growths varied between 1 and 20 m^3/ha/year. This simple model allowed to estimate growth values at a stand level from easy field measurements from forest inventories.

Keywords: biometrics; forest planning; forest growth; stand models; *Nothofagus pumilio*

Tree growth volume is one of the main variables within the forest landscape planning (Vanclay 1994). In the past, estimations were done using fixed values (Piterbarg 1965), and recently individual tree growth equations have been used (Vanclay 1994). The employment of fixed growth values was justified due to the insufficient development of growth models that possess an adequate relationship between data collection costs and outputs precision. However, the actual methodology to estimate stand growth has limitations: (a) different factors that influence tree growth are mixed in one data base, i.e. site quality, crown classes or stand density (e.g. see discussion in Vanclay (1994) and Martínez Pastur et al. (2002)); (b) the models are incompletely developed, i.e. along the full site quality gradient (e.g. Peri, Martínez Pastur 1996); (c) the proposed models have complex methodologies where data collection is not compatible with traditional techniques of forest inventories (e.g. García 1988); and (d) many of the proposed models have incompatibilities with previously developed models, i.e. with individual tree volume equations (e.g. Martínez Pastur, Fernández 1997).

Biometric variables that influence tree growth are innumerable. For this, models must be abstractions of the reality (Hari 1996) that should simplify the complexity of the forest system under modelling (García 1988). The challenge in tree growth modelling resides in the analysis of the forest system and isolation of the main variables that better describe the processes within an acceptable error of estimation. In addition, these variables should be of easy measurement and understanding, and should adapt to a wide range of stand growing conditions (García 1988; Vanclay 1994). One choice is to estimate growth through models developed at a stand level using volume increment/basal area ratios (Martínez Pastur 2006), offering the necessary simplicity in order to make possible their use in the forest planning. The objective of this paper was to develop a simple stand growth model for even-aged pure *Nothofagus pumilio* (Poepp. et Endl) Krasser stands with full density along full site quality and stand age gradients.

MATERIALS AND METHODS

Data collection

Samples were taken in 125 homogeneous, even-aged *Nothofagus pumilio* pure stands with full density, growing at San Justo Ranch (Tierra del Fuego – Argentina) (54°06'S, 68°37'W), along full site quality (IS_{60} = 9.7 to 23.2 m) (Martínez Pastur et al. 1997) and age (30 to 450 years old) gradients. Young stands (less than 120 years old) presented a basal area of 66.5 m^2/ha, while in a maturing growth phase (120 to 250 years) they had 74.6 m^2/ha and in a senescence phase (up to 250 years old) they presented 91.6 m^2/ha. Tree density decreased with stand age and tree diameters, when 6,800 trees/ha were measured in younger stands (600 to 76,000 trees per ha), 760 trees/ha in maturing growth phase stands (250 to 860 trees/ha) and 380 trees/ha in senescence phase stands (250 to 700 trees/ha). For this, plots of variable area were employed for samplings. The area of each plot was determined according to the following requirements: (a) homogeneous and even-aged stands, (b) areas of full density without canopy gaps, (c) the non-inclusion of dead trees in the plots, and (d) including between 18 and 20 trees in each sampling plot. Therefore, the sampled average area varied according to the development growth phase of each stand, being 125 m^2 in young, 300 m^2 in maturing and 400 m^2 in senescence phase stands. In each plot, stand age (average of two dominant individuals) and site quality were determined following the methodology proposed by Martínez Pastur et al. (1997). All trees were sampled with an increment borer, measuring their diameter at breast height (dbh) and discriminated by crown classes (dominant, codominant, intermediate and suppressed). Total over bark volume was estimated using the models and methodologies proposed by Martínez Pastur et al. (2002). Only one core was taken from each tree, being all the extractions oriented to the centre plot. In each core, tree-rings were counted, measuring the 4-year periodic growth during the last 20 years. It was considered that no mortality occurred during this period because no dead trees were found in the plots and the stand growth conditions did not change significantly. Total over bark volume increments were estimated for each plot during the studied period. The analysis included 12,285 age-dbh points of 2,456 trees for all the site quality and age gradients. Final data base corresponded to n = 625 (five pseudo-replicates of 4-year period in each stand).

The proposed model

Few antecedents for volume increment models at a stand level were proposed (e.g. Clutter et al. 1983; Qin, Cao 2006) which use basal area to estimate the growth rate. In this study we used the basal area as a variable, but the other variables of the forest structure at a stand level were analyzed to be included in the proposal of a new model, giving priority to their simplicity, universality and biological attributes.

Site quality, number of trees, quadratic mean diameter, stand mean age, basal area and crown class proportion of the stand trees were analyzed. The selection of the variables was done considering their correlation with the stand growth. Finally, the selected variables to fit the model were site class (*SC*), where a higher increment was expected when the site quality increased, and stand age (*A*), due to a growth potential change along the tree existence over time (Klepac 1976). In order to avoid the stand density effect, the ratio between stand total over bark volume increment (*TVI*) (m^3/ha/year) and stand basal area (*BA*) (m^2/ha) was calculated and used as a dependent variable. A model using this relationship (*Ratio TVI/BA*) was proposed, based on previous growth equations (Martínez Pastur et al. 1997; Peri, Martínez Pastur 1996) where the age of stand was modified exponentially and multiplicatively in function of stand site quality.

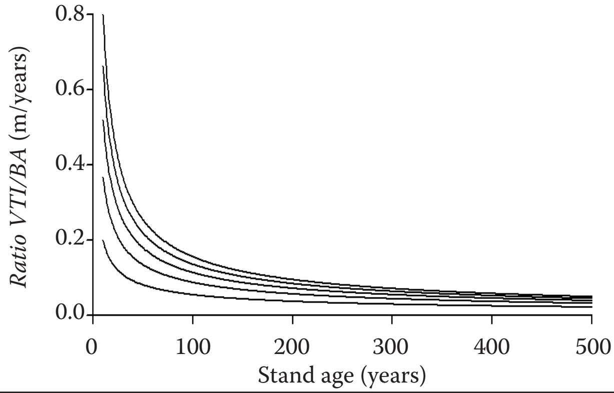
$$Ratio\ TVI/BA = a\,(6 - SC)^b\, A^{(c(6 - SC)^d)}$$

where: *a, b, c, d* – model coefficients,
SC – stand site class in Arabic numbers (1 to 5, *I* to *V*) according to Martínez Pastur et al. (1997),
A – stand age (years).

The model was fitted with non-linear regression and adjusted for *r*-squared (r^2-*adj*), standard error of estimation (*SEE*), mean absolute error (*MAE*) and residual analyses were employed to describe the model adjustment, using average ($\bar{e}$) and absolute ($|\bar{e}|$) errors in site quality and stand age frequencies. Also, the residuals were presented

Table 1. Statistics and parameters of the stand growth model for the total volumetric increment and basal area ratio at a stand level

Parameters	*a*	0.735481
	b	1.070900
	c	–0.564430
	d	0.143966
Statistics	*n*	625
	r^2-*adj.*	0.819
	DRE	0.033
	RP	0.019



DRE – standard error of estimation; *RP* – average residual. *Ratio VTI/BA* – ratio between stand total over bark volume increment and stand basal area. The graph represents site qualities according to Martínez Pastur et al. (1997)

as a percent value of the mean predicted value of each frequency.

$$\bar{e} = \left(\left(\sum_{i=1}^{n} e_i\right) / n\right)$$

$$|\bar{e}| = \left(\left(\sum_{i=1}^{n} |e_i|\right) / n\right)$$

where: n – data number,
e_i – residual (observed value – predicted value),
$\bar{e}$ – average error.

Independent validation

To analyze the fitness and significance of the model, an independent validation was conducted in other stands. For this, data was collected in 18 homogeneous, even-aged *Nothofagus pumilio* pure stands with full density, growing at San Justo Ranch (54°06'S, 68°37'W) and Río Irigoyen forest (54°38'S, 66°37'W) (Tierra del Fuego – Argentina), along a full site quality range (IS_{60} = 9.7 to 23.2 m) and ages between 101 and 305 years old stands. The methodology employed was the same as was previously described. The validation data resulted in n = 90 (five pseudo-replicates of 4-year period in each stand).

RESULTS

Statistics and biological significance of the model

The *TVI/BA* ratio data showed significant variations between site qualities and stand age, mainly in the youngest stands (Fig. 1). This justified their inclusion in the model as independent variables (site quality and stand age). The fitted model presented adequate statistics (Table 1) considering that it covers the full age and site quality gradients for the species. Standard error of estimation and mean

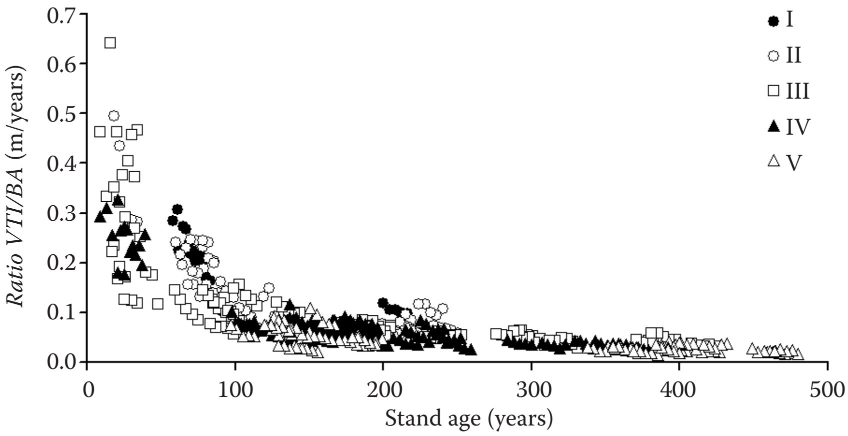


Fig. 1. Data dispersion of the ratio between stand total over bark volume increment and basal area (m/years) along the full site quality gradient. *I* to *V* are site qualities according to Martínez Pastur et al. (1997)

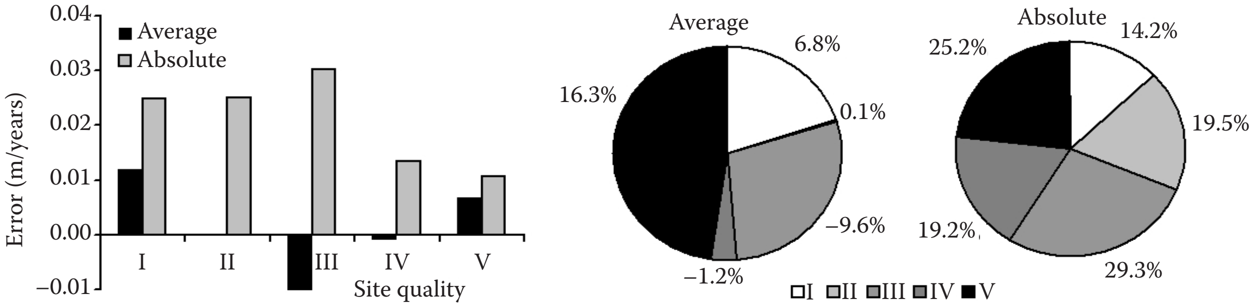


Fig. 2. Average and absolute errors of the model along site quality frequencies. *I* to *V* are site qualities according to Martínez Pastur et al. (1997)

absolute error could be related to the natural data dispersion (Fig. 1). The residual dispersion had a high heterogeneity in the data variances along the stand age gradient. In young stands the data presented a higher dispersion due to *TVI/BA* ratios reaching their maximum values, while in old stands the dependent variable was lower and homogeneous. The proposed model represents the natural data dispersion observed in Fig. 1, maintaining a separation between the curves at different site qualities along the used range (Table 1). Beside this, the separation proportion between the curves incremented according to higher site qualities, increasing the differences when it was diminished.

The ratios between stand total over bark volume increment and stand basal area during the first 40 years of the stand age varied between 0.30 and 0.80 m/year in site quality *I*, and between 0.10 and 0.20 m/year in site quality *V*. In stand ages of 40–80 years, the values of the ratios reached 0.18–0.30 m/year in site quality *I* and 0.06–0.10 m/year in site quality *V*, while in stand ages of 80–200 years the ratio values reached 0.09–0.18 and 0.03–0.06 m/year for site quality *I* and *V*, respectively. In the old stands (up to 200 years old) the ratio values reached 0.05–0.09 and 0.02–0.03 m per year for site quality *I* and *V*, respectively.

Residual analyses were used to describe the error patterns of the model fit. Average values were overestimated in the extreme site qualities (*I* and *V*) and underestimated in the middle site quality (*III*) (Fig. 2). These values represented between 0.1% and 16.3% of the mean predicted value, being higher in the lower site quality. The absolute errors were higher in the upper site classes due to their higher absolute values compared with lower site classes. Despite this, the percentage of the mean predicted values did not significantly change along the site classes (14% to 29%). When the residual errors were analyzed along the age gradient, higher values were observed in young stands (0 to 100 years) (Fig. 3). However, a higher percentage of the mean predicted values occurred in the higher age class (11% in the stand ages up to 300 years). Absolute errors were overestimated in young stands and underestimated in stands with ages over 100 years. Despite this, the percentage of the mean predicted values did not significantly change along the site classes (20% to 23%).

Independent validation of the model

The validation of the model presented an average error of −0.0063 m/year and an absolute error of 0.0132 m/year, which represents 7.3% and 18.9% of the mean predicted value (Table 2). The residual errors were always overestimated in the lower site quality with absolute errors of 0.0146 m/year, which

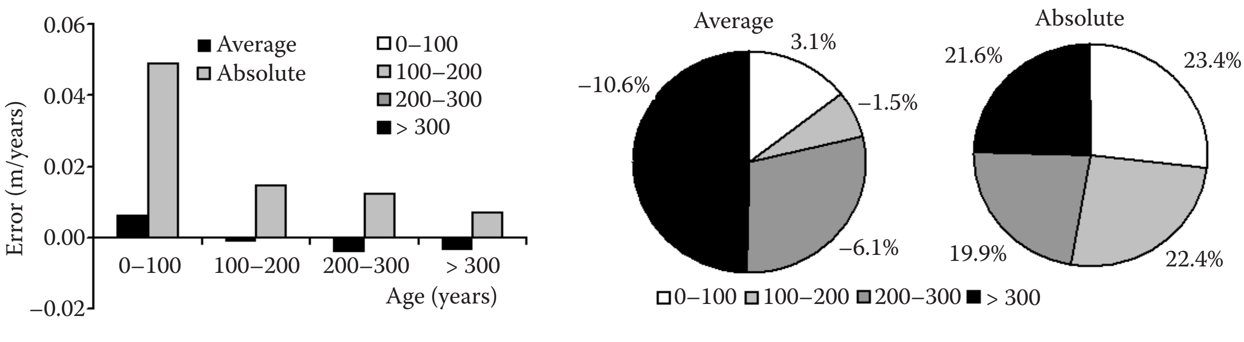


Fig. 3. Average and absolute errors of the model along age frequencies

Table 2. Independent validation of the stand growth model for the total volumetric increment and basal area ratio at a stand level

SC	Stand age (years)	*MPV* (m^3/year)	*AE* (m^3/year)	*ABE* (m^3/year)	*AE*%	*ABE*%
1	156	0.0963	–0.0197	0.0216	–20.5	22.4
2	203	0.0725	–0.0152	0.0152	–21.0	21.0
3	204	0.0713	–0.0024	0.0061	–3.4	8.6
4	138	0.0657	–0.0086	0.0086	–13.0	13.0
5	232	0.0494	0.0146	0.0146	29.5	29.5
Total			–0.0063	0.0132	–5.7	18.9

SC – site qualities according to Martínez Pastur et al. (1997); *MPV* – mean predicted value; *AE* – average error; *ABE* – absolute error; *AE*% – average error as a percent value of the mean predicted value; *ABE*% – average absolute error as a percent value of the mean predicted value

represents 29.5% of the mean predicted value. On the contrary, in the remainder site quality classes (*I* to *IV*) residual errors were underestimated and varied between –0.0024 and –0.0197 m/year, which represents –3.4% and –21.0% of the mean predicted value. Absolute errors varied between 0.0061 and 0.0216 m/year, which represents 8.6% and 22.4% of the mean predicted value.

DISCUSSION

The pseudo-replicates are widely used in growth forest studies (Vanclay 1994; Peri, Martínez Pastur 1996; Martínez Pastur, Fernández 1997), compared to the increment borer samples due to their difficulty in data processing. On the other hand, most of the forest growth studies selected the individual trees to be sampled (Klepac 1976; Hara et al. 1991; Mayor, Roda 1993; Everard, Christie 1995) according to their health and forest structure tree characteristics. This can overestimate the model outputs, because the non-selected trees usually have less growth, e.g. trees with stem rot. For this reason, the sampling of all trees of the stand, as was presented here, allows to achieve more accurate models.

The higher data variation and dispersion were found in young stands with ages less than 100 years, where the tree growth increased exponentially. This stand response is coincident with previous *Nothofagus pumilio* growth studies (Peri, Martínez Pastur 1996; Martínez Pastur 2006), where the higher volumetric increment values are determined by the higher height growth rate of the trees (Martínez Pastur et al. 1997). The lack of variance homogeneity along the stand age gradient is an undesirable condition that is usually found in forest growth data (Martínez Pastur, Fernández 1997). However, this lack was not significant between site qualities when the age classes were assembled. Finally, the independent validation of the model showed a similar trend of the errors than those presented in the residual analysis of the model. The independent data obtained near the sampling for the model adjustment (San Justo Ranch) presented a similar response to those obtained in the fairest sites (Río Irigoyen forest). This is due to the homogeneity in the forest structure of *Nothofagus pumilio* forests along its natural distribution (Martínez Pastur et al. 1994, 2000; Gea et al. 2004).

To use the present model, the obtained *TVI/BA* ratios (m/year) must be multiplied by the stand basal area (m^2/ha) to obtain the total volumetric increment for the stand (m^3/ha/year). If we considered the mean density values reported previously along the full age and site quality gradient for the species (Piterbarg 1965; Schmidt, Urzúa 1982; Martínez Pastur et al. 1994; Gea et al. 2004; Martínez Pastur 2006) in Tierra del Fuego, it was possible to calculate *TVI* values from 1.1 m^3/ha/year in a low site quality and senescence phase stands, to a maximum of 20.0 m^3/ha/year in the higher site quality and young stands with full density stock. Most of the growth values reported in the bibliography are included within this range (Alfonso 1942; Piterbarg 1965; Schmidt, Urzúa 1982; Peri, Martínez Pastur 1996; Martínez Pastur et al. 2001; 2002; Peri et al. 2002), but many of them included large biases, since confounding many variables as site quality or stand age. Another error source could be found in total volume models used to estimate the stand volumetric increments, because it is possible to obtain different outputs using different models (Martínez Pastur 2006). In Patagonia, the most widely used models are local volume equations that introduce overestimations and very substantial underestimations in the individual or stand volume estimation when they are used at a landscape level.

The employment of this kind of stand growth models at a regional level allows to obtain precise forest growth estimations where forest possibility could be defined in volume growth units (m^3/year), as occur in many forests, as well as in Patagonia (Martínez Pastur et al. 2004).

CONCLUSIONS

The present model allows to obtain the volumetric growth values at a stand level using a few easily measurable variables, which were usually measured in the majority of the forest inventories around the world (site quality, basal area and stand age), avoiding the use of the undesirable fixed growth values or complex methodologies included in the individual tree growth models. The proposed model also allows the use of variable diameter plots (Bitterlich 1984) without the need of diameter measurement of all the trees, in a wide range of stand ages and the full site quality gradient. This kind of models could be an effective tool for the ordination planning at a landscape level for the *Nothofagus pumilio* forests, and the proposed methodology could be applied in several species managed in pure and even-aged stands.

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Růstový model porostu na základě přírůstu kruhové výčetní základny

ABSTRAKT: Odhad růstu porostu je velmi důležitý v hospodářské úpravě lesa. V minulosti se obvykle stanovoval srovnáním s danými hodnotami (tabulkami), později pomocí rovnic růstu. Alternativním řešením jsou růstové modely. Cílem příspěvku je vytvořit jednoduchý model pro porost tvořený dřevinou *Nothofagus pumilio*, který je plně zkameněný, s vyjádřením bonity a věku. Měření bylo prováděno ve 125 porostech. Údaje o porostní struktuře a vzorky letokruhových analýz byly použity k odhadu růstu pomocí už existujících biometrických modelů. Růst porostu na zkusných plochách za posledních 20 let byl použit ke kalibraci, kde byl hlavní řídící proměnnou poměr celkového objemového přírůstu k přírůstu na kruhové základně. Vytvořený model stanovuje poměr mezi přírůstem zásoby porostu a kruhovou výčetní základnou jako funkce bonity a věku. Statistické hodnoty (r = 0,819, střední chyba = 0,019 a absolutní střední chyba = 0,033) a analýza reziduí prokázaly vhodnost modelu. Výsledné simulace stanovily objemový přírůst porostu mezi 1 až 20 m^3/ha za rok. Tento jednoduchý model umožňuje stanovit růst porostu na základě jednoduchých měření prováděných při inventarizaci lesa.

Klíčová slova: biometrie; hospodářská úprava lesa; růst porostu; porostní model; *Nothofagus pumilio*

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