Possible methods of Norway spruce (*Picea abies* [L.] Karst.) stem shape description

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ABSTRACT: The paper shows a possibility of using Bookstein coordinates for stem shape studies. Bookstein coordinates are simplified to stem shape diameters, for which tests of multidimensional normality, variance-covariance matrix homogeneity, equality of mean shape vectors and principal component calculation are carried out in sample plots Doubravčice 1 and Štíhlice. Principal components are also calculated for Procrustes tangent coordinates, presented in graphs, and the plots are compared. Doubravčice 1 and Štíhlice plots differ especially in age (70 and 30 years) while they do not differ in tree class representation.

Keywords: Norway spruce (*Picea abies* [L.] Karst.); stem shape; Bookstein coordinates; stem shape diameters; Procrustes tangent coordinates; principal components analysis

Foresters have been studying stem shapes of individual trees for more than 200 years to tabulate stem profiles, volume, assortment, and increment. Prodan (1965) stated that the shape factor theory was formulated for the first time by Paulsen around 1800 and elaborated by Smalian, Klauprecht, Pressler, Hohenadl and later by other authors. Shape quotients were studied by Schiffel around 1897, Hohenadl, Mitscherlich and other modern authors.

All these authors are representatives of so called traditional morphometrics. In recent 25 years, not only "traditional morphometrics" but also "geometrical methods" have developed in biology. Geometric morphometrics is a collection of approaches to the multivariate statistical analysis of Cartesian coordinate data, usually (but not always) limited to landmark point locations. The "geometry" referred to by the word "geometric" is the geometry of Kendall's shape space: the estimation of mean shapes and the description of sample variation of shape using the geometry of Procrustes distance. The multivariate part of geometric morphometrics is usually carried out in a linear tangent space to the non-Euclidean shape space in the vicinity of the mean shape.

More generally, it is the class of morphometric methods that preserve complete information about the relative spatial arrangements of the data throughout an analysis. As such, these methods allow visualisation of group and individual differences, sample variation, and other results in the space of the original specimens.

Kendall's shape space is the fundamental geometric construction, named after David Kendall, underlying geometric morphometrics. Kendall's shape space provides a complete geometric setting for analyses of Procrustes distances between arbitrary sets of landmarks. Each point in this shape space represents the shape of a configuration of points in some Euclidean space, irrespective of size, position, and orientation. In shape space, scatters of points correspond to scatters of entire landmark configurations, not merely scatters of single landmarks. Most multivariate methods of geometric morphometrics are linearisations of statistical analyses of distances and directions in this underlying space.

MATERIAL AND METHODS

The studied material consisted of 191 Norway spruce sample trees from the permanent sample plot Doubravčice 1, felled in 1965, and 191 sample trees from the permanent Norway spruce sam-

Table 1. Characteristics of the examined material, $\overline{S}(X)$ being arithmetic mean of centroid sizes, $\overline{h}$ arithmetic mean of heights, $\overline{d}_{1/10}$ arithmetic mean calculated from the diameters at 1/10 of the stem height

Konšel's tree class	Number of stems	Doubravčice 1			Štíhlice		
		$\overline{S}(X)$ (m)	$\overline{h}$ (m)	$\overline{d}_{1/10}$ (cm)	$\overline{S}(X)$ (m)	$\overline{h}$ (m)	$\overline{d}_{1/10}$ (cm)
1	9	37.15	24.66	25.96	22.83	15.15	16.68
2a	42	34.16	22.67	22.52	19.67	13.05	12.18
2b	51	32.86	21.81	19.02	17.35	11.51	10.88
3	54	28.35	18.81	15.76	16.35	10.85	10.10
4	29	25.51	16.93	14.17	13.69	9.09	8.66
5	6	22.95	15.23	12.83	11.28	7.48	8.05

ple plot Štíhlice, felled in 2002. Both plots belong to the School Forest Enterprise in Kostelec nad Černými lesy. The mean age of the sample trees on Doubravčice plot was 70 years, 30 years on Štíhlice plot. The mean stand diameter d_g on Štíhlice plot was 15.3 cm, the mean stand height h_g was 14.5 m, the top stand height h_{100} was 16.0 m, volume per ha 235 m³, stand density 1.02, site class 32, forest type 3P1. The mean stand diameter d_g on Doubravčice 1 plot was 21.6 cm, the mean stand height h_g was 23.3 m, the top stand height h_{100} was 26.3 m, volume per ha 454.1 m³, stand density 0.99, site class 26, forest type 2K3.

The stems were classed into six Konšel's tree classes. Tree class 1 is composed of dominants (9 individuals), tree class 2a of co-dominant major trees (42 individuals), tree class 2b of co-dominant minor trees (51 individuals), tree class 3 of intermediate trees (54 individuals), tree class 4 of shadegrown vital trees (29 individuals), tree class 5 consists of 6 dying or dead individuals. Mean characteristics of centroid size, height and diameter in 1/10 of the stem height are listed in Table 1.

So-called landmarks were identified on the morphological stem curve. These landmarks originate either at the bottom edge of a stump (approximately in 1/100 of the stem height) and at 1/20 of the stem height and continue by tenths of the stem length symmetrically at its left and right part including the stem top. Each landmark is localised using x (diameter) and y (height) coordinates. These coordinates form the matrix $k \times m$, where k is the number of landmarks and m is the number of dimensions. In our case $k = 23$ and $m = 2$. The matrix is called an *original configuration matrix* and represents a basis for further Procrustes statistic processing.

Bookstein coordinates and stem shape diameters

Bookstein coordinates $(u^B_j, v^B_j)^T, j = 3, ..., k$, (Bookstein 1984, 1986 in Bookstein 1991) are the remaining coordinates of the object after translating,

rotating, and rescaling. The baseline is positioned so that either landmark 1 is sent to (0, 0) and landmark 2 is sent to (1, 0), or to preserve symmetry, baseline landmarks are sent to (−1/2, 0) and (1/2, 0).

Fitting using the baseline is a simple procedure, but it can be criticised on the grounds that the baseline landmarks are chosen subjectively and will be weighted disproportionably in the fitting process. Procrustes fitting treats all landmarks equivalently, avoiding the subjective weighting of two particular landmarks as in baseline fitting.

In our case, the baseline was placed between the mean stem base and the stem top. The coordinates of these two landmarks are (−1/2, 0) and (1/2, 0). Consequently, there were 22 landmarks symmetrically placed on the morphological curve. The first pair is in 1/100 of the stem height and the remaining pairs by the tenths of the stem height. These landmarks are labelled as so-called pseudo-landmarks. They are designed on the organism around the outline between anatomical landmarks.

Bookstein coordinates are calculated in the following way:

$$u^B_j = \{(x_2 - x_1)(x_j - x_1) + (y_2 - y_1)(y_j - y_1)\}/D^2_{12} - \frac{1}{2} \quad (1)$$

$$v^B_j = \{(x_2 - x_1)(y_j - y_1) - (y_2 - y_1)(x_j - x_1)\}/D^2_{12}$$

where $j = 3, ..., k$, $D^2_{12} = (x_2 - x_1)^2 + (y_2 - y_1)^2 > 0$ and $-\infty < u^B_j, v^B_j < \infty$, or equivalently by formula (Mardia 1991 in Dryden, Mardia 1998):

$$\mathbf{U} = c\mathbf{A}(\mathbf{X} - \mathbf{b}) \quad (2)$$

This formula gives us the geometrical interpretation of Bookstein coordinates: $\mathbf{U} = (u^B_j, v^B_j)^T$, $\mathbf{X} = (x_j, y_j)^T$, $j = 3, ..., k$, $c > 0$ is the scale, $\mathbf{A}$ is the $m \times m$ rotation matrix, where we rotate by α radians, $\mathbf{b}$ is the translation vector.

Example: Consider a 10 m high stem of the diameter $d_{1/10} = 0.05$ m at 1/10 of its height. Then the coordinates of the mean stem basis are $\mathbf{X}_1 = (0, 0)^T$

and the stem top $X_2 = (10. 0)^T$. These two landmarks compose a baseline with the coordinates of $(-1/2, 0)^T$ and $(1/2, 0)^T$. Bookstein coordinates of the landmark on the morphological stem curve at 1/10 of its height $X_3 = (1, 0.05)^T$ are:

$$\begin{pmatrix} u^B_3 \\ v^B_3 \end{pmatrix} = \frac{1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \left\{ \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} - \right.$$

$$\left. - \begin{pmatrix} (x_2 - x_1)/2 \\ (y_2 - y_1)/2 \end{pmatrix} \right\}$$

where $\alpha = \arctan\{(y_2 - y_1)/(x_2 - x_1)\}$, thus

$$\begin{pmatrix} u^B_3 \\ v^B_3 \end{pmatrix} = \frac{1}{\sqrt{(10 - 0)^2 + (0 - 0)^2}} \begin{pmatrix} \cos 0 & \sin 0 \\ -\sin 0 & \cos 0 \end{pmatrix} \left\{ \begin{pmatrix} 1 \\ 0.05 \end{pmatrix} \right.$$

$$\left. - \begin{pmatrix} (10 - 0)/2 \\ (0 - 0)/2 \end{pmatrix} \right\} = \begin{pmatrix} -0.4 \\ 0.005 \end{pmatrix}$$

If the x-coordinates of morphological landmarks are placed at relative distances (in our case by 1/10 of the stem height), u^B_j assumes the same values in all stems as well. Furthermore, let us consider the stem being symmetrical by the vertical axis, which would simplify the situation as it would limit the number of variables. The stem can be described as a multidimensional object by means of "stem shape diameters". Thus, the stem shape diameters (b_m) are the diameters at the relative sections (in our case $m = 1/100, 1/20, 1/10, ..., 9/10$ of the stem height), divided by the stem height (h), therefore

$$b_m = d_m/h$$

Dividing by the height is in fact elimination of the size from the object in the sense of intuitive definition of the shape. In the case of Procrustes coordinates, the size is eliminated by dividing by the centroid size.

Centroid size is the square root of the sum of squared distances of a set of landmarks from their centroid, or, equivalently, the square root of the sum of the variances of the landmarks about that centroid in x- and y-directions. Centroid size is used in geometric morphometrics because it is approximately uncorrelated with every shape variable when landmarks are distributed around mean positions by independent noise of the same small variance at every landmark and in every direction.

The *centroid size* is given by

$$S(X) = \sqrt{\sum_{i=1}^{k} \sum_{j=1}^{m} (X_{ij} - \overline{X}_j)^2} \tag{3}$$

where X_{ij} is the (i, j)-th entry of X and the arithmetic mean of the j-th dimension is

$$\overline{X}_j = \frac{1}{k} \sum_{i=1}^{k} X_{ij}$$

Note: The stem height does not necessarily have to be used as a baseline; it can be e.g. the diameter in 1/100 of the stem height or another diameter in the lower part of the stem. In that case, we obtain shape quotients and shape series.

The values of Spearman coefficient of correlation of dependence between centroid size and diameters at 1/100, 1/20, 1/10, ..., 9/10 of the stem height for stems from Doubravčice plot are as follows: 0.77, 0.85, 0.86, 0.86, 0.86, 0.86, 0.86, 0.85, 0.82, 0.77, 0.72.

All coefficients of correlation are statistically significant on the level of 0.05.

In dendrometry, ideal column volume is often used to eliminate the stem size in 3D space. The values of Spearman coefficient of correlation of dependence between centroid size and volumes of ideal columns with their bases at 1/100, 1/20, 1/10, ..., 9/10 of stem height are as follows: 0.78, 0.85, 0.86, 0.87, 0,86, 0.87, 0.86, 0.86, 0.84, 0.81, 0.78.

All coefficients of correlation are statistically significant on the level of 0.05. It is evident that there is a strong relation between the values of centroid size and "traditional" quantities.

Individual stem is therefore taken as one pick from n objects described by m dimensions (stem shape diameters ($b_{i, j}$)). Hence: $b_i = (b_{i, 1}, ..., b_{i, m})^T$, $i = 1, ..., n$.

For this selection, it is possible to set a sample vector for mean values $\hat{\mu}$ given by the following equation:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} b_i \tag{4}$$

Estimation of the variance-covariance matrix is ruled by the following equation:

$$S = \frac{1}{n-1} \sum_{i=1}^{n} (b_i - \hat{\mu})(b_i - \hat{\mu})^T \tag{5}$$

The test of hypothesis that the data are derived from multidimensional normal distribution

In this paper, we use a test based on multidimensional skewness ($g_{1, m}$) and kurtosis ($g_{2, m}$) as described in MELOUN and MILITKÝ (1998). We test simultaneous validity of a hypothesis about symmetry ($H_{01}: g_{1, m} = 0$) and about normality of kurtosis ($H_{02}: g_{2, m} = m(m + 2)$) distribution variable of examination. The estimation of sample skewness is given by the following equation:

$$\hat{g}_{1, m} = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} d^3_{ij} \tag{6}$$

where $d_{ij} = (\boldsymbol{b}_i - \hat{\boldsymbol{\mu}})^T \boldsymbol{S}^{-1} (\boldsymbol{b}_j - \hat{\boldsymbol{\mu}})$ is squared Mahalanobis distance. Considering the H_{01} hypothesis as valid, then the test statistics

$$U_1 = \frac{n}{6} \hat{g}_{1,m} \qquad (7)$$

has asymptotically chi-square distribution $\chi^2_{m(m+1)(m+2)/6}$.

The estimation of sample kurtosis is given by the following equation:

$$\hat{g}_{2,m} = \frac{1}{n^2} \sum_{i=1}^{n} d^2_{ii} \qquad (8)$$

Considering the H_{02} hypothesis as valid, then the test statistics:

$$U_2 = (\hat{g}_{2,m} - g_{2,m}) / (8m((m+2)/n))^{0.5} \qquad (9)$$

has asymptotically normal distribution N(0, 1). This approximation can be used providing the following condition is fulfilled:

$$\hat{g}_{2,m} > m(m+2)(n-1)/(n+1) \qquad (10)$$

The test of hypothesis that the mean vectors are equal

Consider two independent random samples $\boldsymbol{b}_{x,1}, ..., \boldsymbol{b}_{x,n}$ (from sample plot Doubravčice) and $\boldsymbol{b}_{y,1}, ..., \boldsymbol{b}_{y,n}$ (from sample plot Štíhlice). The vectors $\boldsymbol{b}_i = (b_1, ..., b_k)^T$, $i = 1, ..., n$ are stem shape diameters. In our case $k = 11$ and $n = 191$. We expect that stems from these populations have mean shapes $\boldsymbol{\mu}_x$ and $\boldsymbol{\mu}_y$.

The test of hypothesis about mean vector equality ($H_0 : \boldsymbol{\mu}_x = \boldsymbol{\mu}_y$ versus $H_1 : \boldsymbol{\mu}_x \neq \boldsymbol{\mu}_y$) can be carried out using Hotelling's T^2 two-sample test. Let us use the following test statistics:

$$F_{stat} = \frac{n_1 n_2 (n_1 + n_2 - m - 1)}{(n_1 + n_2)(n_1 + n_2 - 2)m} (\boldsymbol{\mu}_x - \boldsymbol{\mu}_y)^T \boldsymbol{S}_p^{-1} (\boldsymbol{\mu}_x - \boldsymbol{\mu}_y) \quad (11)$$

where

$$S_p = \frac{(n_1 - 1) S_1 + (n_2 - 1) S_2}{n_1 + n_2 - 2}$$

is the pooled variance-covariance matrix and $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are variance-covariance matrices for individual samples.

Provided the null hypothesis is valid, the test statistics F_{stat} has Fisher's distribution with m and $n_1 + n_2 - m - 1$ degrees of freedom. However, this test can be used only in the case of normality of both sets and homogeneity of variance-covariance matrices.

RESULTS AND DISCUSSION

In the case of Doubravčice plot, the sample skewness is $g_{1,11} = 27.52$. The test statistics U_1 thus equals 876, which is more than the critical value of $\chi^2_{286}(0.05) = 326.4$. Sample kurtosis is $g_{2,11} = 184.6$. Test statistics $U_2 = 17.0$ and the critical value of standardised normal distribution on the significance level of 0.05 is 1.64. The criterion (10) is fulfilled as $g_{2,m} > 141.5$. In both quantities, skewness and kurtosis, we therefore reject the coincidence with normal distribution.

It also applies to the case of Štíhlice plot. Sample skewness is $g_{1,11} = 61.03$ and test statistics $U_1 = 1,942.8$. Sample kurtosis is $g_{2,11} = 235.83$ and test statistics $U_2 = 37.92$. The criterion (10) is fulfilled again ($g_{2,m} > 141.5$). In this case, we also reject coincidence with the normal distribution.

All tests carried out indicate a divergence from multidimensional normal distribution in both sets.

Subsequently, the Box's M test on homogeneity of variance-covariance matrices, MARDIA et al. (1979) was carried out. We reject the hypothesis about ho-

Table 2. Spearman coefficient of correlation for the dependence between stem shape diameters for all the stems from Doubravčice 1 and Štíhlice plots. All coefficients of correlation are significant on the level of 0.05

|	$b_{0.9}$	$b_{0.8}$	$b_{0.7}$	$b_{0.6}$	$b_{0.5}$	$b_{0.4}$	$b_{0.3}$	$b_{0.2}$	$b_{0.1}$	$b_{0.05}$	$b_{0.01}$
$b_{0.9}$	1	0.65	0.57	0.54	0.51	0.47	0.43	0.42	0.39	0.39	0.36
$b_{0.8}$	0.65	1	0.84	0.81	0.72	0.67	0.60	0.55	0.49	0.46	0.51
$b_{0.7}$	0.57	0.84	1	0.91	0.85	0.79	0.72	0.67	0.60	0.55	0.59
$b_{0.6}$	0.54	0.81	0.91	1	0.92	0.87	0.81	0.77	0.70	0.66	0.64
$b_{0.5}$	0.51	0.72	0.85	0.92	1	0.95	0.91	0.86	0.79	0.76	0.65
$b_{0.4}$	0.47	0.67	0.79	0.87	0.95	1	0.96	0.93	0.88	0.85	0.61
$b_{0.3}$	0.43	0.60	0.72	0.81	0.91	0.96	1	0.96	0.92	0.90	0.56
$b_{0.2}$	0.42	0.55	0.67	0.77	0.86	0.93	0.96	1	0.95	0.93	0.54
$b_{0.1}$	0.39	0.49	0.60	0.70	0.79	0.88	0.92	0.95	1	0.96	0.52
$b_{0.05}$	0.39	0.46	0.55	0.66	0.76	0.85	0.90	0.93	0.96	1	0.49
$b_{0.01}$	0.36	0.51	0.59	0.64	0.65	0.61	0.56	0.54	0.52	0.49	1
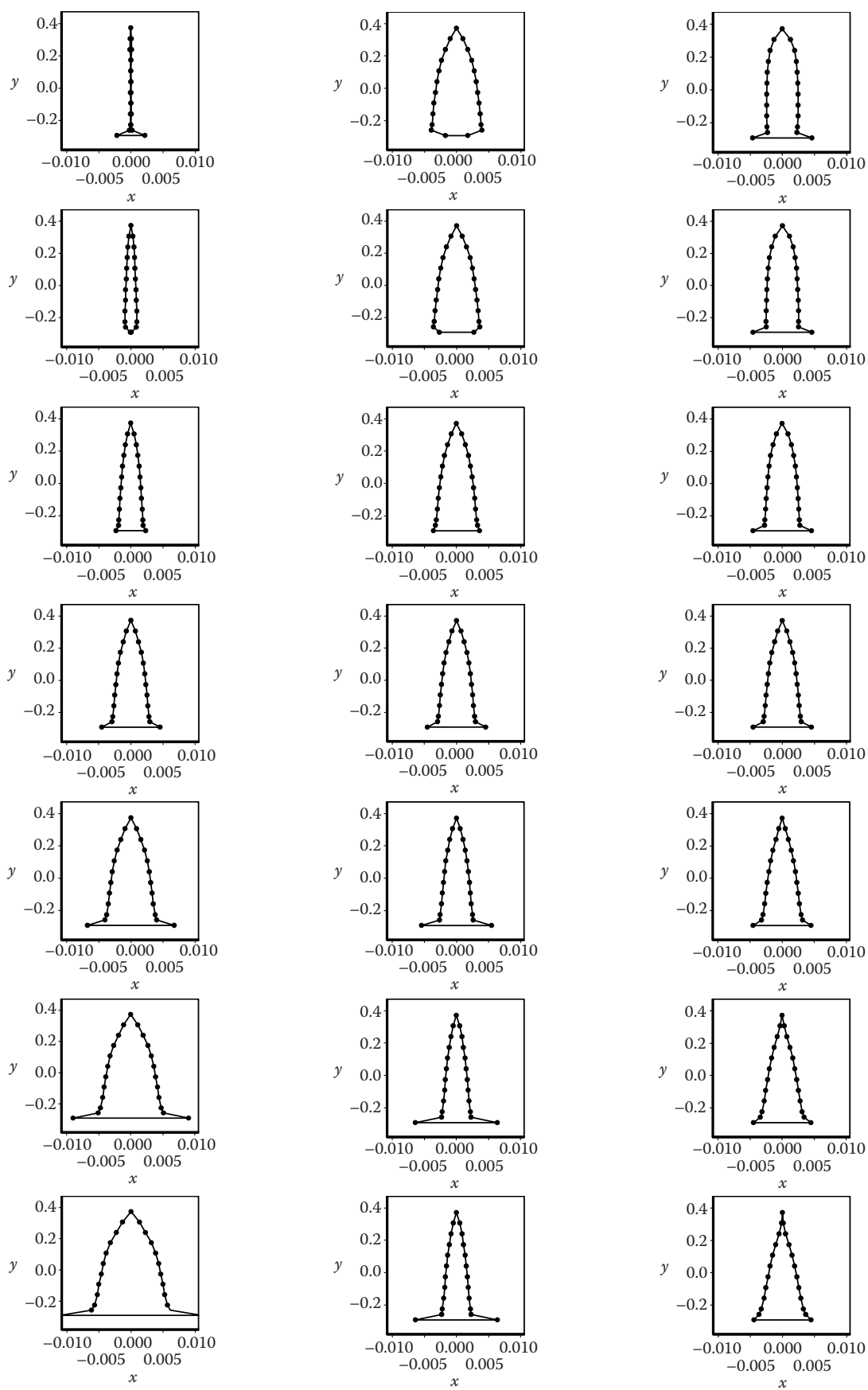
Fig. 1. The columns show the shapes at −3, −2, −1, 0, 1, 2, 3, standard deviations along the first three PC for Doubravčice 1 plot
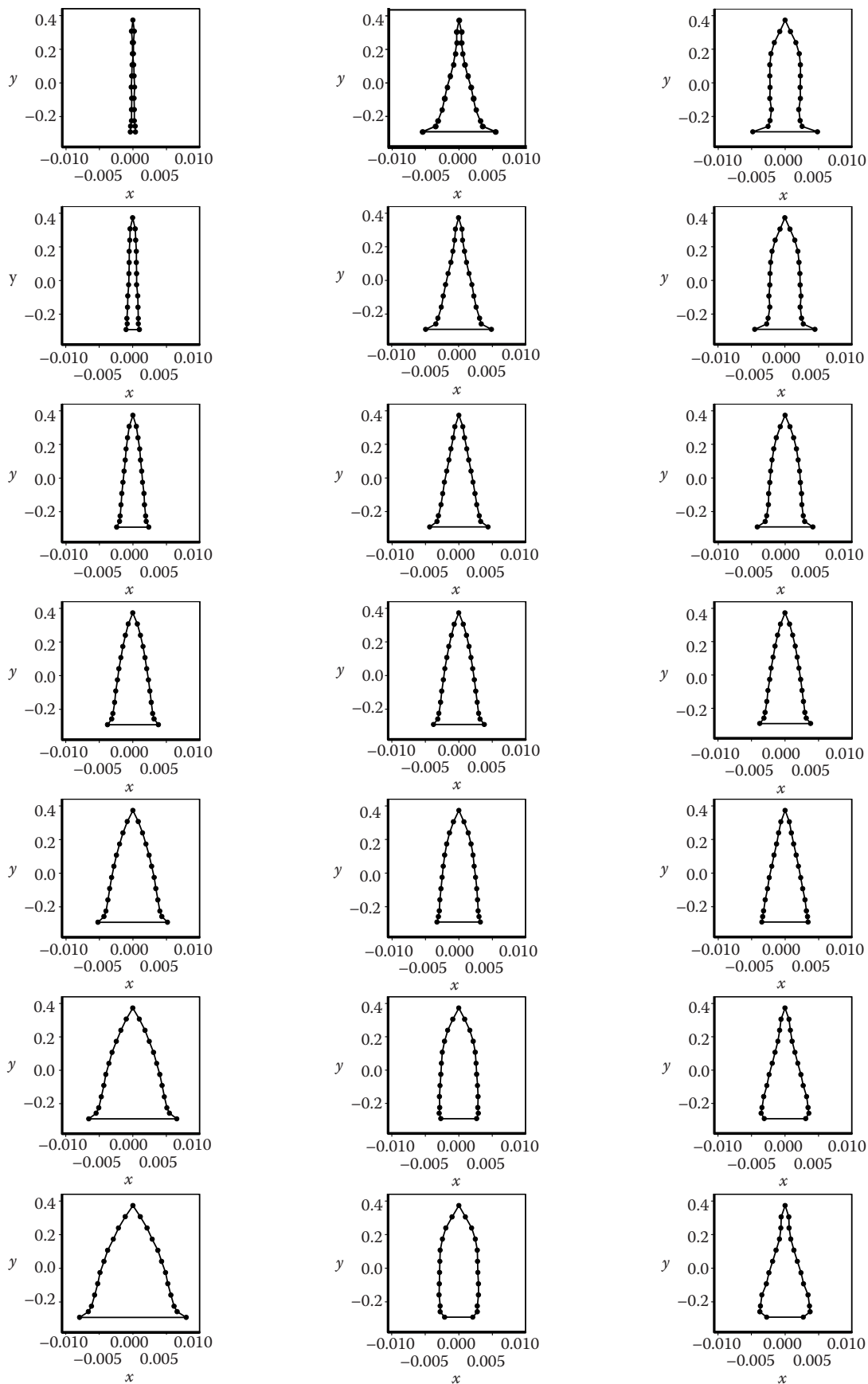
Fig. 2. The columns show the shapes at −3, −2, −1, 0, 1, 2, 3, standard deviations along the first three PC for Štíhlice plot

mogeneity of variance-covariance matrices on the significance level of 0.05.

The assumption of normality and equal covariances turned out to be questionable. Therefore, Monte Carlo test was carried out with the null hypothesis that the groups have equal mean shapes. The data were randomly split into two groups of the same size as the groups in the data, and the test statistic F_{stat} was evaluated for B random permutations $T_1, ..., T_b$. The ranking r of the observed test statistic F_{obs} was then used to give the p-value of the test:

$$p\text{-value} = 1 - \frac{r - 1}{B + 1}$$

For each pair of locations, 2,000 random permutations were performed. P-value is less than 0.01, therefore we reject the hypothesis about the equality of mean shapes vectors.

Variability

The *principal components analysis* (PCA) was used to analyse the shape variability. The prerequisite for the use of PCA method is a higher rate of linear dependence between the variables. Table 2 determines the correlation matrix for stem shape diameters (b_m) for all stems from Doubravčice 1 and Štíhlice plots. The table shows that the prerequisite of a higher rate of linear dependence between stem shape diameters is fulfilled.

The orthogonal eigenvectors of variance-covariance matrix, denoted by χ_i, $i = 1, 2, ..., j$, are the *principal components* of variance-covariance matrix with corresponding eigenvalues

$$\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_j \geq 0$$

where: $j = \min (n - 1, m)$.

The summary of the first three principal components for Doubravčice 1 and Štíhlice plots, calculated from the stem shape diameters and from Procrustes

tangent coordinates is shown in Table 3. Eigenvalues in Doubravčice 1 plot are the same for both the stem shape diameters and Procrustes tangent coordinates and almost the same in Štíhlice plot.

Graphic effect of the first three principal components is the same in stem shape diameters as well as in Procrustes tangent coordinates. For better illustration and with regard to already prepared programmes by DRYDEN (2000), we carried out an analysis of the first three principal components in Procrustes tangent coordinates as shown in DRYDEN and MARDIA (1998), the definitions having been introduced in the same way as in KŘEPELA et al. (2004).

Figs. 1, 2, 3 and 4 show the graphic effect of the first three principal components in both sets. The first principal component explains approximately 83% variability in both sets. It has the same graphic effect in both sets and coincides with the first principal components in the previous papers of KŘEPELA et al. (2001, 2004), KŘEPELA (2002), for the Norway spruce and Scotch pine. The other two principal components in both sets do not have the same graphic effect. If looking for differences in shapes between the two sets, we can proceed in the frame of scores of the individual principal components. Let us express the squared Mahalanobis distance D^2 of equation:

$$F_{stat1} = \frac{n_1 n_2 (n_1 + n_2 - M - 1)}{(n_1 + n_2)\,(n_1 + n_2 - 2)M}\,D^2 \qquad (12)$$

where $M = (k - 1)m - m/2\,(m - 1) - 1$ is the dimension of shape space, k is the number of landmarks, m is the real dimension of object, as:

$$D^2 = \sum_{j=1}^{M} s_j^2/\lambda_j$$

where $s_j = (\bar{v} - \bar{w})^T \chi_j$ are the scores in the direction of the observed group difference, $\bar{v}$, $\bar{w}$ are the sample means of Procrustes tangent coordinates.

Table 3. Eigenvalues of the variance-covariance matrices of stem shape diameters and Procrustes tangent coordinates from Doubravčice 1 and Štíhlice plots, and proportional expression of variability explained by them

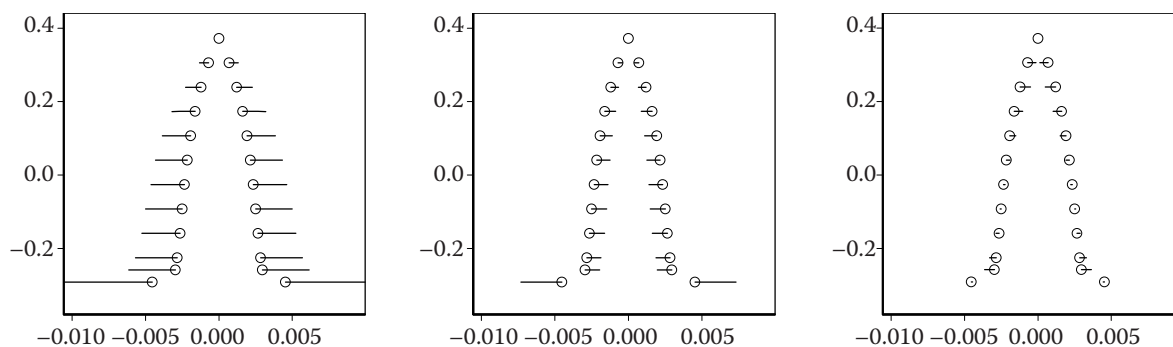| Eigenvalue | Doubravčice 1 | | | | Štíhlice | | | |
| | Stem shape diameters | | Procrustes tangent coordinates | | Stem shape diameters | | Procrustes tangent coordinates | |
	$\lambda_j . 10^{-6}$	$\lambda_j / \sum_{j=1}^{p} \lambda_j . 100$ (%)	$\lambda_j . 10^{-3}$	$\lambda_j / \sum_{j=1}^{p} \lambda_j . 100$ (%)	$\lambda_j . 10^{-6}$	$\lambda_j / \sum_{j=1}^{p} \lambda_j . 100$ (%)	$\lambda_j . 10^{-3}$	$\lambda_j / \sum_{j=1}^{p} \lambda_j . 100$ (%)
λ_1	10.488	83.5	1.5198	83.5	8.3487	83.2	1.0002	82.9
λ_2	1.6842	13.4	0.6090	13.4	0.7421	7.4	0.3196	8.5
λ_3	0.2364	1.9	0.2282	1.9	0.3884	3.9	0.2072	3.6

Fig. 3. The first three PC with configurations evaluated at 3 standard deviations along each PC from the full Procrustes mean in Doubravčice 1 plot
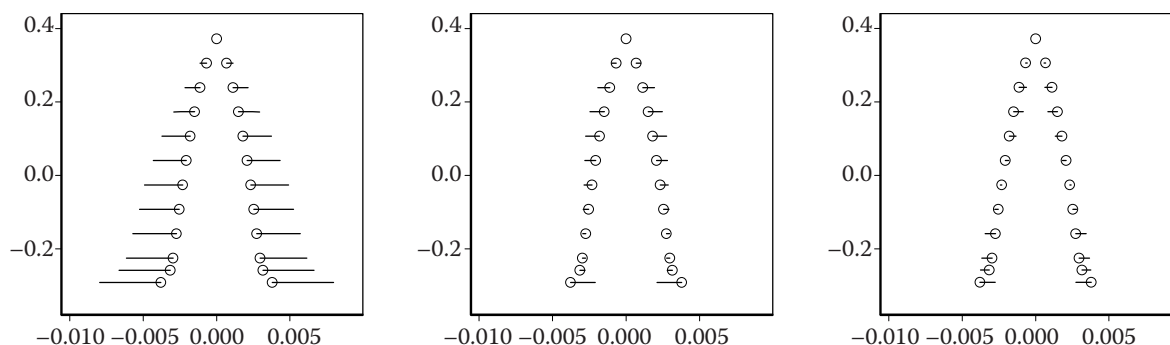

Fig. 4. The first three PC with configurations evaluated at 3 standard deviations along each PC from the full Procrustes mean in Štíhlice plot

High values of s^2_j/λ_j indicate which directions of shape variability are associated with the difference between the groups. Table 4 shows the values of F_{stat1} from Equation (12) calculated for the first 15 principal components. The values of F_{stat1} for further principal components are less than 0.01.

It results from the table that principal components No. 3, 2 and 4 contribute most to the difference between the sets.

The second principal component in Doubravčice 1 plot explains 13.4% of total variability. It concerns the effect of "buttresses". The variability issues from the robust (thin) buttresses compared to the thin (robust) rest of the stem. The third principal component explains 1.9% of the total variability. It concerns the stems with damaged tops.

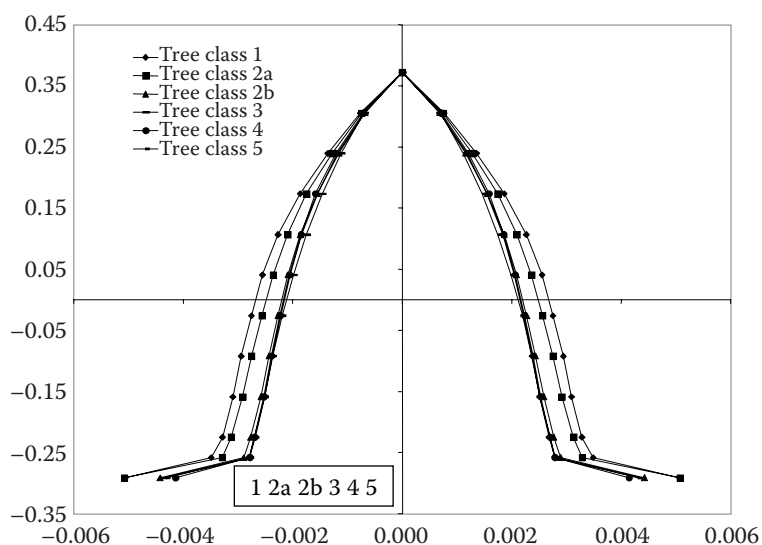In Štíhlice plot, the second principal component (explaining 8.5% variability) is the asymmetrical

Fig. 5. Courses of full Procrustes mean shapes for single Konšel's tree classes in Doubravčice 1 plot
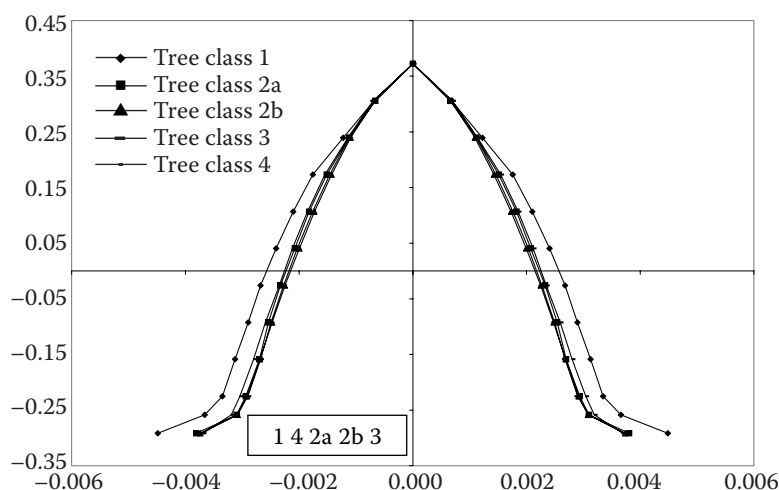
Fig. 6. Courses of full Procrustes mean shapes for single Konšel's tree classes in Štíhlice plot
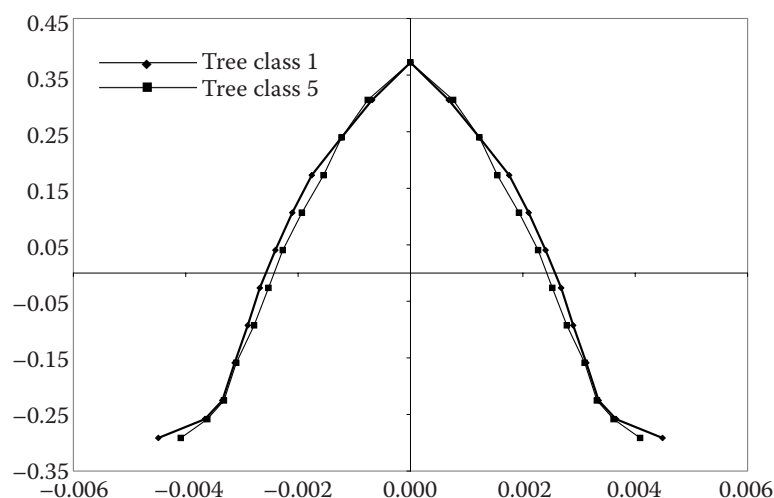

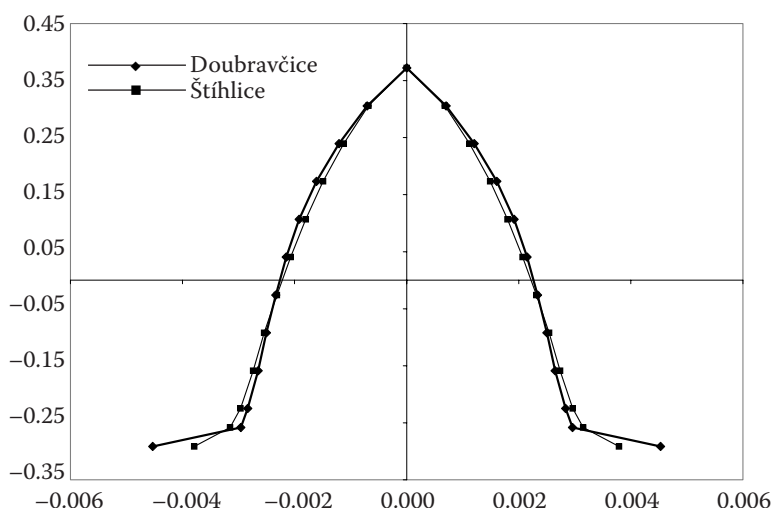Fig. 7. Courses of full Procrustes mean shapes for Konšel's tree classes 1 and 5 in Štíhlice plot

Fig. 8. Courses of full Procrustes mean shapes for Doubravčice 1 and Štíhlice plots

graphic effect. In the lower part of the stem (up to 1/10h), the direction of this effect is reversed in comparison with the remaining upper part of the stem. The second principal component might be explained from the biological aspect by rot in the lower part of the stem caused by *Armillaria ostoyae* (Romagn.) Herink.

The graphic effect of the third principal component (3.6% variability) is rather complicated. In the lower part of the stem (1/100h) and subsequently at 5/10h..., 8/10h the effect has one direction while at 1/20h, 1/10h, 2/10h, 3/10h the direction is reversed. So far, no biological explanation has been found.

Table 4. F_{start1} partition of Equation (12) for the first 15 principal components

No. of principal component	F_{start1} partition				
1–5	0.2785	5.0200	7.8300	1.2200	0.0000
6–10	0.0584	0.0612	0.1260	0.4900	0.1300
11–15	0.4470	0.0000	0.1600	0.1090	0.0097

Fig. 5 shows full Procrustes mean shapes for single Konšel's tree classes in Doubravčice 1 plot. The shapes can be divided into three groups. The first group consists of Konšel's tree class 1, the second of Konšel's tree class 2a, and the third of the remaining Konšel's tree classes. The same situation was presented in Křepela et al. (2001) for the whole area of Doubravčice 1 (484 stems). Here, we made a selection of 191 stems. The selection was adjusted to Štíhlice plot as regards the tree class representation. In Fig. 6, we see again full Procrustes mean shapes for single Konšel's tree classes in Štíhlice plot. Konšel's tree class 1 constitutes a unique group. The remaining tree classes have not shown the shape differentiation so far. In tree class 4, we see swelling of the lower part of the stem caused by rot. Tree class 5 shows particularly unique shapes. All the individual trees were attacked by the fungus *Armillaria ostoyae* (Romagn.) Herink. Shape comparison of this tree class with tree class 1 is shown in Fig. 7.

When comparing Doubravčice 1 and Štíhlice plots, we in fact compare a 70-year old tree stand and a 30-year old one, with the Konšel's tree classes equally represented in both stands, and the growth conditions being similar. Our study brought evidence that both plots differ in their mean shapes. Štíhlice plot (Fig. 8) has thinner buttresses; the stems are wider up to 4/10 of the height and thinner in the upper part of the stem. The stem shape in Štíhlice plot was influenced by a more extensive fungal attack of tree class 4 and 5.

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Některé možnosti popisu tvaru kmene smrku ztepilého (*Picea abies* [L.] Karst.)

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ABSTRAKT: Článek se zabývá možností využití Booksteinových souřadnic při studiu tvaru kmene, Booksteinovy souřadnice jsou zjednodušeny do „tvarových kmenových průměrů" a pro ně je v rámci souborů Doubravčice 1 a Štíhlice proveden test vícerozměrné normality, shody variančně-kovariančních matic, test rovnosti vektorů středních hodnot

a výpočet hlavních komponent. Hlavní komponenty jsou vypočteny také pro Procrustovy tangentové souřadnice a je provedeno jejich grafické znázornění a porovnání mezi soubory. Soubory Doubravčice 1 a Štíhlice se liší především ve stáří (70 a 30 let), neliší se v zastoupení stromových tříd.

Klíčová slova: smrk ztepilý (*Picea abies* [L.] Karst.); tvar kmene; Booksteinovy souřadnice; tvarové kmenové průměry; Procrustovy tangentové souřadnice; analýza hlavních komponent

Booksteinovy souřadnice jsou jednou z tzv. „geometrických metod" popisu tvaru biologických i technických objektů. Objekt (v našem případě kmen) je popsán pomocí hraničních bodů umístěných na morfologické křivce, jejichž x-ové a y-ové souřadnice vytvářejí základní konfigurační matice jednotlivých kmenů. Hraniční body byly umístěny do 1/100, 1/20, 1/10, …, 9/10 výšky kmene. Jako základní úsečka (a baseline) byla použita výška kmene. Střed báze kmene má tedy souřadnice (−1/2, 0) a vrchol kmene (1/2, 0). Geometricky je kmen ve vodorovné pozici a základní úsečka prochází osou kmene a má velikost rovnu 1. Booksteinovy souřadnice jednotlivých hraničních bodů $(u^B_j, v^B_j)^T$, j= 3, …, k, potom zaručují otočení, posunutí a přeškálování základních konfiguračních matic s ohledem na základní úsečku, která je pro všechny kmeny stejná. V našem případě máme zjednodušenou situaci, neboť uvažujeme kmen jako symetrické těleso (průměr je určen jako aritmetický průměr dvou kolmo na sebe měřených průměrů) a navíc výšky postupují po relativních vzdálenostech, takže pro všechny kmeny souřadnice u^B_j nabývá stejných hodnot. Tím můžeme přejít k „tvarovým kmenovým průměrům" – b_m. Jde vlastně o průměry v relativních sekcích vydělené výškou kmene. Výška kmene se tedy používá k odstranění velikosti z objektu. V Procrustově analýze se k odstranění velikosti používá centrální velikost. V klasické lesnické dendrometrii se k odstranění velikosti používají průměry buď v relativních, nebo v absolutních výškách (tvarové kvocienty, tvarové řady) a dále objemy ideálních válců při výpočtu výtvarnic.

Pomocí tvarových kmenových průměrů bylo popsáno všech 191 kmenů v souboru Doubravčice 1 i v souboru Štíhlice. Oba srovnávané soubory pocházejí z podobných růstových podmínek na ŠLP Kostelec nad Černými lesy, mají stejné zastoupení stromových tříd a jejich hlavní odlišnost představuje věk (70 a 30 let).

Cílem bylo provést test rovnosti vektorů středních hodnot tvarových kmenových průměrů mezi oběma soubory a provést rozbor variability.

Klasický test rovnosti středních tvarových vektorů je parametrický. Předem tedy bylo nutné ověřit normalitu dat a homogenitu variančně-kovariančních matic. K tomuto účelu jsme použili testy založené na vícerozměrné šikmosti a špičatosti, resp. Boxův M-test. Jak předpoklad normality, tak předpoklad homogenity variančně-kovariančních matic byly zamítnuty. K porovnání rovnosti vektorů středních hodnot jsme tedy použili Monte Carlo test, který prokázal, že oba soubory se liší ve vektoru svých středních hodnot.

Dále byla provedena analýza hlavních komponent pro tvarové kmenové průměry i pro Procrustovy tangentové souřadnice. Výsledek je obsažen v tab. 3. Procento variability vysvětlené prvními třemi hlavními komponentami se liší, ne však zásadně, pouze u souboru Štíhlice. Grafický efekt prvních tří hlavních komponent je stejný jak pro kmenové tvarové průměry, tak pro Procrustovy tangentové souřadnice. Proto jsme dále použili Procrustovy tangentové souřadnice (vzhledem k lepší názornosti a také již ke zhotoveným funkcím ve statistickém programu S-plus).

Obě první hlavní komponenty v obou souborech vysvětlují přibližně 83 % variability. Mají symetrický grafický efekt po celé délce kmene (obr. 1–4) a tento efekt je totožný s efektem „největších" hlavních komponent v předchozích článcích autorů – Křepela et al. (2001, 2004), Křepela (2002) u smrku i u borovice. Druhá hlavní komponenta v souboru Doubravčice 1 vysvětluje 13,4 % celkové variability. Jde o efekt „kořenových náběhů". Variabilita pochází ze silných (úzkých) kořenových náběhů oproti úzkému (silnému) zbytku kmene. Třetí hlavní komponenta vysvětluje 1,9 % celkové variability. Jde o kmeny s poškozeným vrcholem.

V souboru Štíhlice má druhá hlavní komponenta (vysvětluje 8,5 % variability) asymetrický grafický efekt. Ve spodní části kmene (do 1/10h) je směr tohoto efektu opačný než ve zbývající horní části kmene. Biologické vysvětlení pro druhou hlavní komponentu by bylo možné hledat v hnilobě spodní části kmene způsobené václavkou *Armillaria ostoyae* (Romagn.) Herink. Třetí hlavní komponenta (3,6 % variability) má grafický efekt dosti komplikovaný. Ve spodní části kmene (1/100h) a dále v 5/10h, …, 8/10h má tento efekt jeden směr a v 1/20h, 1/10h, 2/10h,

3/10*h* směr opačný. Biologické vysvětlení pro tuto komponentu se nepodařilo nalézt.

Největší tvarové rozdíly mezi oběma soubory je nutno hledat v rámci variability vysvětlené 3., 2. a 4. hlavní komponentou, jak udává tab. 4.

Obr. 5 a 6 znázorňují tvarové průměry získané z plných Procrustových souřadnic pro jednotlivé Konšelovy stromové třídy. V souboru Doubravčice 1 vidíme tři skupiny tvarů. První tvoří stromy předrůstavé, druhou skupinu pak stromy úrovňové hlavní a poslední skupinu vrůstavé ustupující stromy, zastíněné životaschopné stromy a stromy odumírající a odumřelé. V souboru Štíhlice jsou skupiny dvě. První je tvořena předrůstavými stromy, druhá pak ostatními stromovými třídami, mezi nimiž ještě nedošlo k tvarové diferenciaci. Stromová třída 4 byla napadena hnilobou – proto došlo ke tvarovému zbytnění ve spodní části kmene.

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