Diameter and height growth analysis for individual White Pine trees in the area of Kostelec nad Černými lesy

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ABSTRACT: Four theoretical growth functions, Mitscherlich, Logistic, Gompertz and Korf functions, were applied to the growth data on dbh and tree height of individual Eastern White Pine (*Pinus strobus*) trees to model their growth. The current increments, mean increments and relative growth rates of dbh and height as functions of age were given by derivation from the best fit growth functions. The growth processes of dbh and height were divided into three periods by the inflexion points of the corresponding current increment curves.

Keywords: Eastern White Pine (*Pinus strobus* L.); growth analysis; Gompertz function; Korf function; Logistic function; Mitscherlich function

The Eastern White Pine (*Pinus strobus* L.) is a native of eastern North America, where it grows from Georgia northward as far as Newfoundland. It forms both mixed and pure stands on moist clayey to clayey-sandy soils. In its native country it can reach a height of 40 m and as much as 150 cm in diameter. It is moderately resistant to smoke pollution, and its long flexible needles are particularly ornamental. The species was brought to Europe in 1705, and today it is widely planted in forest plantations and in parks. In this paper Mitscherlich, Logistic, Gompertz and Korf functions were used to model the dbh (diameter at breast height) and height growth of four average trees chosen from an even-aged Eastern White Pine stand in Kostelec nad Černými lesy of the Czech Republic, the best fit growth functions were selected, the current increments, mean increments and relative growth rates of dbh and height as functions of age were given by derivation from the best fit growth functions, and the growth processes of dbh and height of the trees were analyzed.

MATERIAL AND METHODS

The Eastern White Pine stand is located near the town of Kostelec nad Černými lesy about 35 km E of Prague. The geographical position: longitude 14°51´E and latitude 50°01´N. Mean annual temperature, mean temperature of January and mean temperature of July are 8.14°C, –1.92°C and 17.82°C, respectively. The highest temperature (12. 7. 1991) is 40.8°C and the lowest (8. 1. 1985) is –28.5°C. Mean annual precipitation is 662.60 mm. The geological bottom is Permian and chalk sandstone covered with a thick layer of loam loess. According to silvicultural classification, the area is classified as acid beech-oak forest (Roček et al. 1998).

A 0.1 ha research plot with 120 trees was established in a uniform single-species Eastern White Pine stand. Diameter at breast height and tree height as well as crown class were recorded for each tree on the plot. Four trees whose dbh were closest to the quadratic mean diameter of the plot were chosen as sample trees. The sample trees were felled and a complete stem analysis (Avery, Burkhart 1983) was made for each sample tree. The average dbh and height of the 4 sample trees was used for the growth analysis each year.

The sample trees were severed 30 cm above the ground to minimize the effects of butt swell and every sample tree was cut into uniform 1-meter sections, except the final top section. The measurement of annual rings was made to the

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nearest 0.01 mm with CODIMA (an increment measuring device equipped with a microscope). The age of the trees counted at breast height was 26 years.

Programs EXCEL, STATGRAPHICS and SAS were used for data processing.

Fit of growth functions

Mitscherlich, Logistic, Gompertz and Korf growth functions were fitted to the growth data of dbh and height:

Mitscherlich $$y = A(1 - e^{-bt}) \quad (1)$$

Logistic $$y = \frac{A}{1 + e^{a - bt}} \quad (2)$$

Gompertz $$y = Ae^{-e^{a - bt}} \quad (3)$$

Korf $$y = Ae^{\frac{b}{(1-a)t^{a-1}}} \quad (4)$$

where: y – the dbh (or height) at age t,
e – the base of natural logarithms,
a, b – the parameters to be determined by the method of "least squares" fitting for the growth data,
A – asymptotic dbh (height) – the point where dbh (height) growth equals zero and it was estimated by Yuan Zhifa's "three points" method.

Mitscherlich curve is characterized by not having any inflexion with asymptote at $f(t) = A$ while Logistic, Gompertz and Korf curves are sigmoid curves with asymptotes at $f(t) = 0$ and $f(t) = A$. Logistic and Gompertz curves are characterized by an inflexion occurring exactly at the halfway and approximately at one-third points of the entire growth process, respectively (SWEDA, KOIDE 1981).

The goodness of fit was evaluated by R^2 or MSSD (SWEDA, KOUKETSU 1984):

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - y_i)^2}{\sum_{i=1}^{n}(Y_i - \overline{Y})^2} \quad (5)$$

$$\text{MSSD} = \frac{1}{n - f}\sum_{i=1}^{n}(Y_i - y_i)^2 \quad (6)$$

where: R^2 – coefficient of determination,
MSSD – mean squared sum of deviations,
Y_i – observed value at age i,
y_i – calculated value at age i,
$\overline{Y}$ – average of the actual observation,
n – total age of the sample tree,
f – number of parameters involved in the equation concerned, i.e. two for Mitscherlich and three for the others.

The denominator $n - f$ in the above expression ensures a fair comparison of growth equations with different number of parameters.

Analysis of growth process

Current annual increment and relative growth rate

Differentiating the best fit growth function, $y = f(t)$, with respect to age gives current annual increment G and dividing G by the existing y gives relative growth rate R (the ratio of increment of size to size itself), that is

$$G = \frac{dy}{dt} = f'(t) \quad (7)$$

$$R = \frac{G}{y} = \frac{f'(t)}{f(t)} \quad (8)$$

If the second derivative of growth function at age t_1 is 0, the current annual increment G gets its maximum value, i.e. when $f''(t_1) = 0$, $G = f'(t_1) = \max$.

The mean relative growth rate $\overline{R}$ (CAUSTON 1981) during the interval $t_j - t_i$ is

$$\overline{R} = \frac{\ln Y_j - \ln Y_i}{t_j - t_i} \quad (9)$$

For Korf growth function

$$G = \frac{Ab}{t^a} \cdot e^{b(1-a)^{-1} t^{1-a}} \quad (10)$$

its maximum value

$$G_{\max} = \frac{Aa}{e} \cdot \sqrt[a-1]{\frac{a}{eb}} \quad (11)$$

$$t_1 = \sqrt[a-1]{\frac{b}{a}} \quad (12)$$

$$R = \frac{b}{t^a} \quad (13)$$

Mean annual increment (MAI) and its culmination age

$$\text{MAI} = \frac{y}{t} = \frac{f(t)}{t} \quad (14)$$

The maximum of mean annual increment is at age t_2, when the first derivative of the function of mean annual increment is 0 and the mean annual increment equals the current annual increment.

For Korf function

$$\text{MAI} = \frac{A}{t} e^{b(1-a)^{-1} t^{1-a}} \quad (15)$$

$$t_2 = \sqrt[a-1]{b} \quad (16)$$

Division of the growth process

Inflexion points of the current annual increment curve are at age t_{31} and t_{32}, at which the third derivative of the growth function equals zero (i.e. $y''' = 0$) and the growth rates change most rapidly. The growth process is divided into three periods (LI, MEGH 1993): $[0, t_{31})$, $[t_{31}, t_{32})$ and $[t_{32}, +\infty)$.

For Korf function

$$t_{31}, t_{32} = \sqrt[a-1]{b \cdot \frac{3\,\mu\,\sqrt{5-\frac{4}{a}}}{2\,(a+1)}} \qquad (17)$$

In the first period, $[0, t_{31})$, the growth rate is lower and the growth rate curve is concave. This period can be called a "pre-rapid" growth period. The second period, $[t_{31}, t_{32})$, with the highest growth rate can be called a rapid growth period in which the growth rate curve appears convex. During the third period, $[t_{32}, +\infty)$, the growth rate gets lower and the growth rate curve is concave, so the period can be called a "post-rapid" growth period.

RESULTS AND DISCUSSION

Analysis of dbh growth

Growth functions

Mitscherlich, Logistic, Gompertz and Korf functions were applied to the dbh growth data of the Eastern White Pine sample trees. The parameters for each growth function and the fit statistics are listed in Table 1, and the curves of these growth functions are shown in Figs. 1–4 respectively.

As seen in Table 1, Korf growth function achieved the best fit with the highest R^2 and the smallest MSSD, followed by Gompertz function, Logistic function and Mitscherlich function in the sequence of fit goodness.

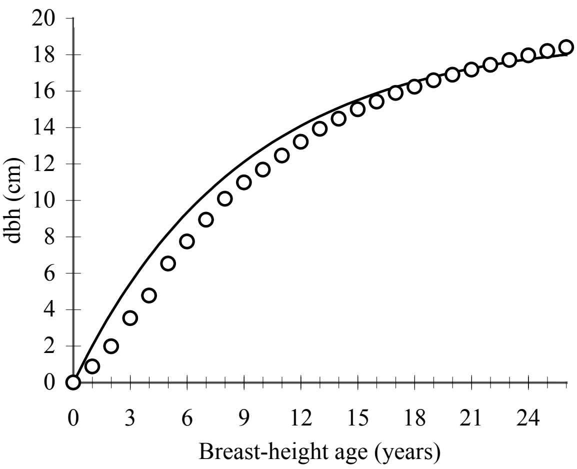

Fig. 1. Mitscherlich curve (line) compared with the observed dbh growth (dots)

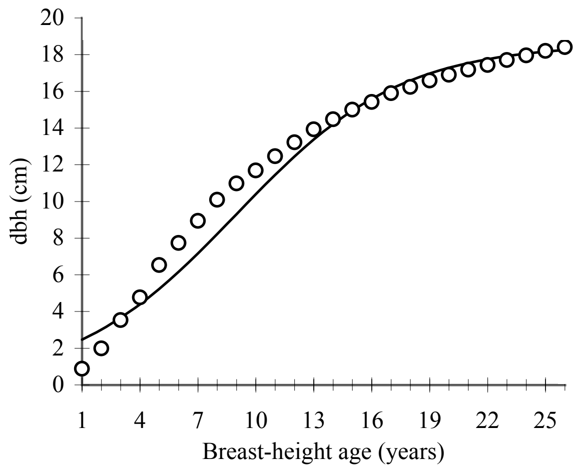

Fig. 2. Logistic curve (line) compared with the observed dbh growth (dots)

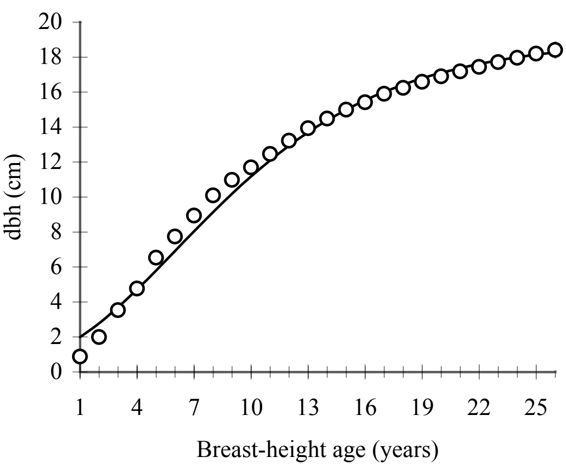

Fig. 3. Gompertz curve (line) compared with the observed dbh growth (dots)

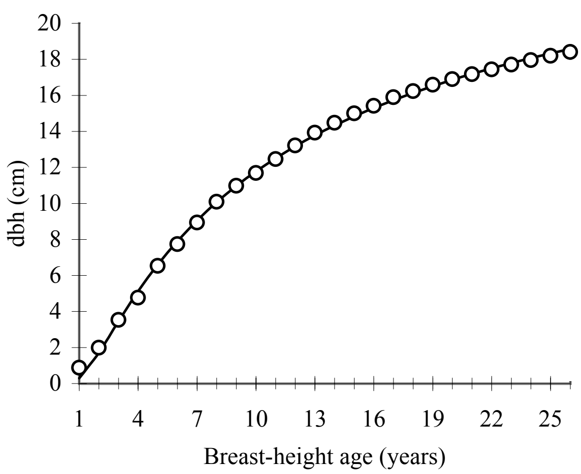

Fig. 4. Korf curve (line) compared with the observed dbh growth (dots)

Table 1. Parameters of the growth functions and the indicators of fit

Type	a	b	A	Growth function	R^2	MSSD
Mitscherlich		0.1125	19.0287	$y_{dbh} = 19.0287\,(1 - e^{-0.1125t})$	0.9677	1.1704
Logistic	2.1105	0.2349	18.5876	$y_{dbh} = \frac{18.5876}{1 + e^{2.1105 - 0.2349t}}$	0.9697	0.9470
Gompertz	0.9749	0.1606	19.0287	$y_{dbh} = 19.0287e^{-e^{0.9749 - 0.1606t}}$	0.9914	0.2700
Korf	1.6921	3.2055	30.1982	$y_{dbh} = 30.1982e^{\frac{-3.2055}{0.6921t^{0.6921}}}$	0.9989	0.0351

Current annual increment, mean annual increment and relative growth rate

The first derivative of the fittest growth function was used to express the current annual increment of dbh (G_{dbh}), i.e.

$$G_{dbh} = \frac{96.7997}{t^{1.6921}} \cdot e^{-4.6318t^{-0.6921}} \tag{18}$$

From equation (11) and equation (12), the maximum of current annual increment (G_{max-d}) and the culmination age (t_{1-d}) were calculated as follows:

$$G_{max-d} = \frac{Aa}{e} \cdot \sqrt[a-1]{\frac{a}{eb}} = 1.760 \qquad \text{(cm)}$$

$$t_{1-d} = \sqrt[a-1]{\frac{b}{a}} = 2.52 \qquad \text{(years)}$$

that means at the age of 3 years (breast height age) the current annual increment of dbh got the maximum value (1.760 cm).

Substituting the parameters in equations (15) and (16) by the corresponding values from Table 1, the mean annual increment of dbh as a function of age was given by:

$$\text{MAI}_{\text{dbh}} = \frac{30.1982}{t} \cdot e^{-4.6318t^{-0.6921}} \tag{19}$$

and its culmination age (t_{2-d}) was

$$t_{2-d} = \sqrt[a-1]{b} = 5.38 \qquad \text{(years)}$$

The curves of equation (18) and equation (19) are shown in Fig. 5. As can be seen in Fig. 5, the two curves crossed at the age 6, and at the same time the mean annual increment of dbh got the maximum value (1.323 cm).

Relative growth rate of dbh as a function of age was

$$R_{dbh} = \frac{3.2055}{t^{1.6921}} \tag{20}$$

The curve of function (20) is shown in Fig. 6. It can be seen that the gradient of the curve was negative. The relative growth rate decreased rapidly at an early stage, then the decrease was slower.

The mean relative growth rate of dbh at age 1 to 26 was

$$\bar{R}_{dbh} = \frac{\ln 18.41 - \ln 0.88}{26 - 1} = 0.122$$

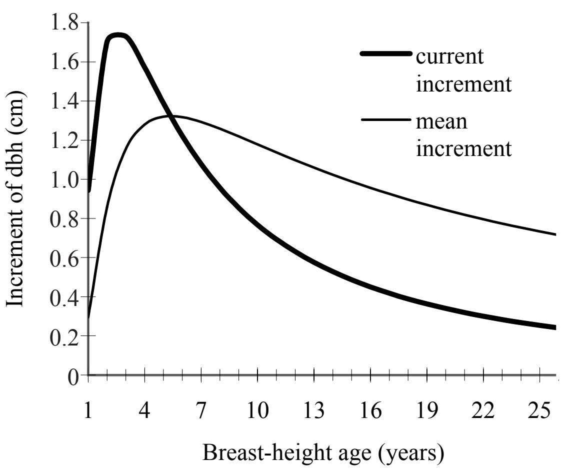

Fig. 5. The curves of current annual increment and mean annual increment of dbh

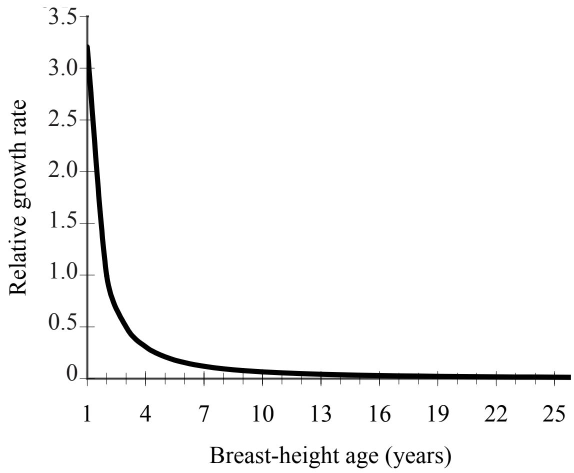

Fig. 6. The relative growth rate of dbh as a function of age

Table 2. Parameters of the growth functions and the indicators of fit

Type	a	b	A	Growth function	R²	MSSD
Mitscherlich		0.0596	23.6379	$y_h = 23.6379\,(1 - e^{-0.0596t})$	0.8969	5.2964
Logistic	3.3035	0.2307	21.2588	$y_h = \dfrac{21.2588}{1 + e^{3.3035 - 0.2307t}}$	0.9776	1.0912
Gompertz	1.3949	0.1173	23.6379	$y_h = 23.6379e^{-e^{1.3949 - 0.1173t}}$	0.9957	0.2115
Korf	1.5084	4.0537	88.9473	$y_h = 88.9473e^{\frac{-4.0537}{0.5084t^{0.5084}}}$	0.9983	0.0815

Division of the growth process of dbh

Substituting *a* and *b* in equation (17) by the corresponding values from Table 1, the age t_{31-d} and t_{32-d}, at which the inflexion points of the current annual increment curve appeared, would be

$t_{31-d} = 0.75$ (years)

$t_{32-d} = 4.32$ (years)

That means the "pre-rapid" growth period of dbh was very short – in the first year (breast height age) dbh of the trees started their rapid growth. Therefore, the growth process of dbh could be divided into the following two periods: 0 to 4 years and 5 years up. During the first period the average growth rate of dbh was

$$\frac{f(t_{32-d})}{t_{32-d}} = 1.280 \text{ cm per year.}$$

Analysis of tree height growth

Growth functions

The four growth functions mentioned above were fitted to the growth data of tree height. The parameters for each growth function and the indicators of fit are given in Table 2, and the curves of these growth functions are shown in Figs. 7–10.

It can be seen from Table 2 that Korf function fitted the height growth data with highest accuracy, followed by Gompertz function and by Logistic function while Mitscherlich function fitted the height growth with the least accuracy.

Current annual increment, mean annual increment and relative growth rate

Substituting the parameters in equation (10), (11) and (12) by the corresponding values from Table 2, the current annual increment of tree height as a function of age would be

$$G_h = \frac{360.5645}{t^{1.5084}} \,.\, e^{-7.9730t^{-0.5084}} \quad (21)$$

and its maximum value (G_{max-h}) and culmination age (t_{1-h}) would be

$G_{max-h} = 0.99$ (m)

$t_{1-h} = 6.99$ (years)

In other words, the current annual increment of tree height was maximum (0.99 m) at the age 7.

According to equation (15), the mean annual increment of tree height as a function of age is given by

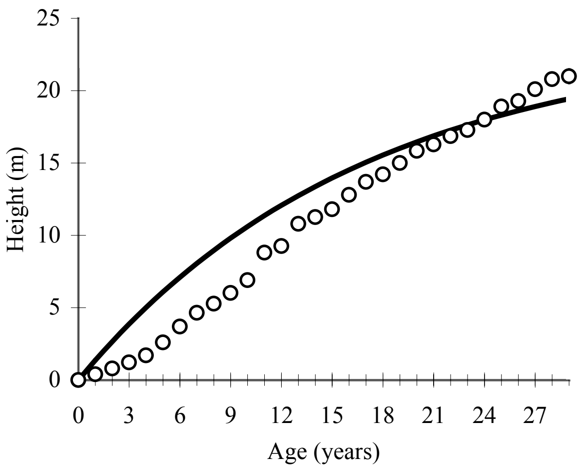

Fig. 7. Mitscherlich curve (line) compared with the observed height growth (dots)

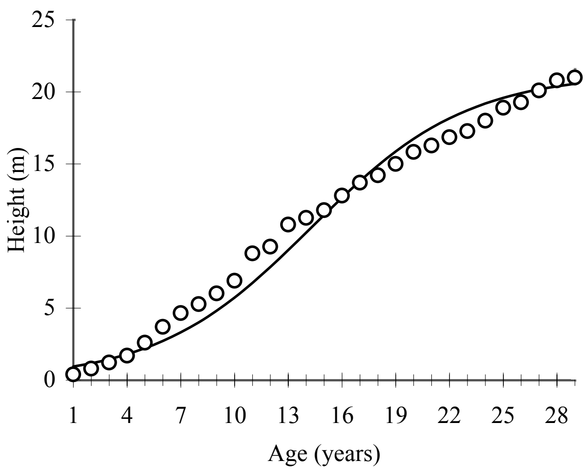

Fig. 8. Logistic curve (line) compared with the observed height growth (dots)

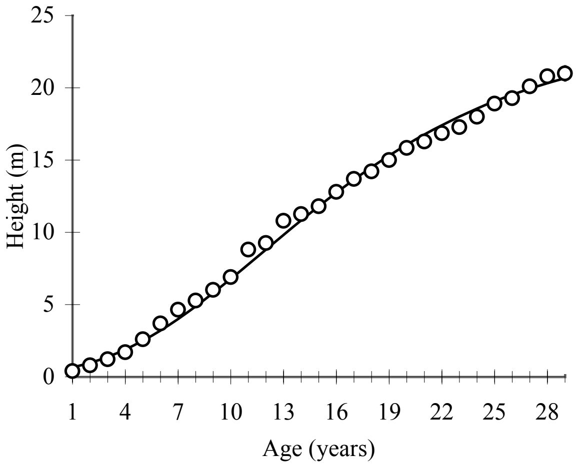

Fig. 9. Gompertz curve (line) compared with the observed height growth (dots)

Fig. 10. Korf curve (line) compared with the observed height growth (dots)

$$\text{MAI}_h = \frac{88.9473}{t} \cdot e^{-7.9730 t^{-0.5084}} \qquad (22)$$

The curves of equations (21) and (22) are shown in Fig. 11. As shown in Fig. 11, the curve of mean annual increment crossed the curve of the current annual increment at the age 16 (t_{2-h} = 15.69), meanwhile the mean annual increment of tree height got its maximum value (0.79 m).

According to equation (13), relative growth rate of tree height as a function of age was

$$R_h = \frac{4.0537}{t^{1.5084}} \qquad (23)$$

The curve of the relative growth rate of tree height is shown in Fig. 12. The figure shows that the relative growth rate decreased rapidly before 4 years old and then the decrease got slower and slower.

The mean relative growth rate of height from age 1 to 29 was

$$\overline{R}_h = \frac{\ln 21 - \ln 0.40}{29 - 1} = 0.141$$

Division of the growth process of tree height

According to equation (17), age t_{31-h} and age t_{32-h} for the current annual increment curve of tree height would be

$$t_{31-h} = 1.40 \qquad \text{(years)}$$

$$t_{32-h} = 12.85 \qquad \text{(years)}$$

Thus, the height growth process was divided into the following three periods: 0 to 1 years, 2 to 12 years and 13 years up. In the rapid growth period (2 to 12 years), the average increment of tree height per year was

$$\frac{f(t_{32-h}) - f(t_{31-h})}{t_{32-h} - t_{31-h}} = 0.90 \qquad \text{(m)}$$

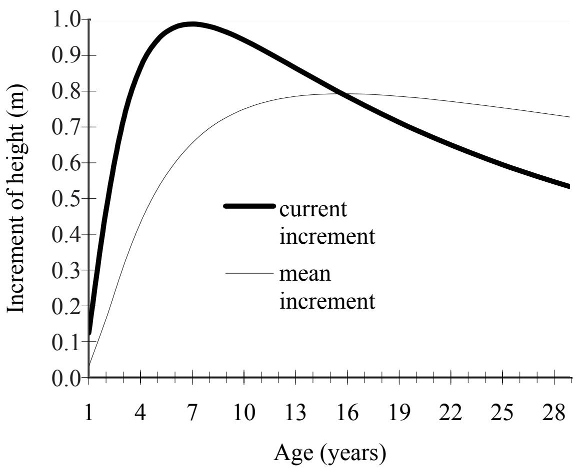

Fig. 11. The curves of current annual increment and mean annual increment of height

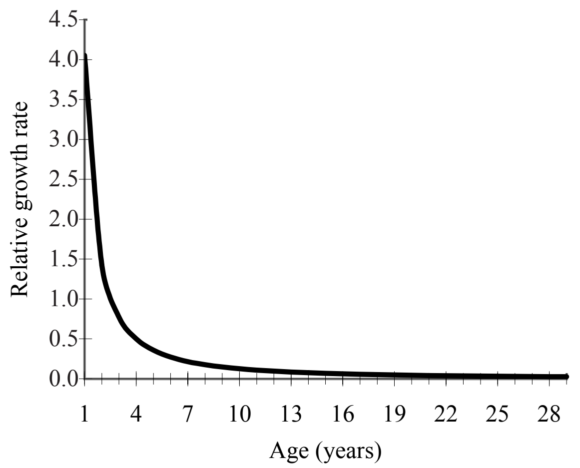

Fig. 12. The relative growth rate of height as a function of age

CONCLUSIONS

The application of Mitscherlich, Logistic, Gompertz and Korf functions to the dbh and height growth data of the individual Eastern White Pine trees revealed that Korf function fitted the dbh and height growth best and expressed the growth processes very well, followed by Gompertz function and by Logistic function, and Mitscherlich function fitted the dbh and height growth with the least accuracy.

The current increments, mean increments and relative growth rates of dbh and height as functions of age, which were derived from the best fit growth functions, were used for growth analysis. The maximum values of the current annual increment of dbh and height were 1.760 cm and 0.99 m, and occurred at the age of 6 (breast height age was 3) and 7, respectively. The maximum values of the mean annual increment of dbh and height were 1.323 cm and 0.79 m, and occurred at the age of 9 (breast height age was 6) and 16, respectively. The curves of relative growth rate of dbh and height with respect to age showed a similar trend, and the average relative growth rates of dbh and height were 0.122 and 0.141, respectively.

There are different ways to divide growth processes. In this paper, the two inflexion points of the current annual increment curve were used to divide growth process into three periods: "pre-rapid", "rapid" and "post-rapid" growth periods. The "pre-rapid" growth period of dbh was very short, so the growth process of dbh was divided into rapid growth period (breast height age 0 to 4 years) and "post-rapid" growth period (5 years up). The three periods for height growth were as follows: 0 to 1 years, 2 to 12 years and 13 years up. During the rapid growth periods the average increments per year of dbh and height were 1.280 cm and 0.90 m, respectively.

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Analýza růstu výčetní tloušťky a výšky středního kmene borovice vejmutovky na jedné z lokalit na území ŠLP Kostelec nad Černými lesy

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ABSTRAKT: Příspěvek uvádí výsledky analýzy tloušťkového a výškového růstu středního kmene stejnorodého a stejnověkého porostu borovice vejmutovky (*Pinus strobus* L.) na jedné z lokalit na území ŠLP Kostelec nad Černými lesy. K analýze byly využity čtyři standardní růstové funkce: Mitscherlichova, logistická, Gompertzova a Korfova. Byl sledován běžný přírůst studovaných růstových veličin, průměrný přírůst a relativní růstová intenzita jako funkce věku pro nejvíce vyhovující růstovou funkci. Jako taková byla prokázána funkce Korfova. Růstový proces bylo možné rozdělit do tří výrazných růstových období na základě různé dynamiky růstu.

Klíčová slova: borovice vejmutovka; růstová analýza; Gompertzova funkce; Korfova funkce; logistická funkce; Mitscherlichova funkce

Příspěvek uvádí výsledky analýzy tloušťkového a výškového růstu středního kmene stejnorodého a stejnověkého porostu borovice vejmutovky (*Pinus strobus* L.) na jedné z lokalit na území ŠLP Kostelec nad Černými lesy. Stanoviště je charakterizováno SLT 2K, průměrnou roční teplotou 8,14 °C a průměrnými ročními srážkami 660 mm. K analýze byly použity čtyři standardní růstové funkce: Mitscherlichova, logistická, Gompertzova a Korfova. Byl sledován běžný přírůst studovaných růstových veličin (výčetní průměr a výška středního kmene), průměrný přírůst a relativní růstová intenzita jako funkce věku pro nejvíce vyhovující růstovou funkci. Jako taková byla prokázána funkce Korfova, dále následovaly funkce Gompertzova, logistická a konečně Mitscherlichova. Maximální hodnoty běžného přírůstu výčetního průměru a výšky byly 1,76 cm a 0,99 m, byly zaznamenány ve věku šesti a sedmi let. Maximální hodnoty průměrného přírůstu výčetní tloušťky a výšky byly 1,323 cm a 0,79 m a byly doloženy ve věku devíti a 16 let. Růstový proces bylo možné rozdělit do tří výrazných růstových období na základě dynamiky růstu: iniciální, juvenilní stadium pomalého růstu, velice krátké, dále období rychlého růstu (do čtyř let) a období pomalejšího růstu od pěti let věku (výčetní tloušťka). Podobná období pro výškový růst byla 0–1 rok, 2–12 let a od 12 let výše. Během období rychlého růstu dosahovaly přírůstky výčetní tloušťky a výšky průměrně 1,28 cm a 0,90 m.

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