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Conservation of forest through provision of alternative sources of income; evidence from rural households in Northern Pakistan

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Electronic Supplementary Material (ESM)

The authors are fully responsible for both the content and the formal aspects of the electronic supplementary material. No editorial adjustments were made.

ESM 1.

Comparative analysis

Total differentiation of Equation (10) and (11) yields the following results:

$$\left[\frac{\partial I_a}{\partial P_f}\ \frac{\partial I_a}{\partial P_i}\ \frac{\partial L_f}{\partial P_f}\ \frac{\partial L_f}{\partial P_i}\quad \frac{\partial I_a}{\partial P_a}\quad \frac{\partial L_f}{\partial P_a}\quad \frac{\partial I_a}{\partial IN_s}\quad \frac{\partial L_f}{\partial IN_s}\quad \frac{\partial I_a}{\partial E_a}\ \frac{\partial I_a}{\partial A_a}\ \frac{\partial L_f}{\partial E_a}\ \frac{\partial L_f}{\partial A_a}\quad \frac{\partial I_a}{\partial W}\ \frac{\partial I_a}{\partial \lambda}\ \frac{\partial L_f}{\partial W}\ \frac{\partial L_f}{\partial \lambda}\right] =$$

$$-\left[\frac{\partial F_1}{\partial I_a}\frac{\partial F_1}{\partial L_f}\ \frac{\partial F_2}{\partial I_a}\frac{\partial F_2}{\partial L_f}\right]^{-1}\left[\frac{\partial F_1}{\partial P_f}\ \frac{\partial F_1}{\partial P_i}\ \frac{\partial F_2}{\partial P_f}\ \frac{\partial F_2}{\partial P_i}\quad \frac{\partial F_1}{\partial P_a}\quad \frac{\partial F_2}{\partial P_a}\quad \frac{\partial F_1}{\partial IN_s}\quad \frac{\partial F_2}{\partial IN_s}\quad \frac{\partial F_1}{\partial E_a}\ \frac{\partial F_1}{\partial A_a}\ \frac{\partial F_2}{\partial E_a}\ \frac{\partial F_2}{\partial A_a}\quad \frac{\partial F_1}{\partial W}\ \frac{\partial F_1}{\partial \lambda}\ \frac{\partial F_2}{\partial W}\ \frac{\partial F_2}{\partial \lambda}\right]$$

$$X = \left[Y_{aI_a}\ 0 - Y_{aL_a}\ Y_{fL_f}\quad -10\ 0-1\quad P_aY_{aI_aA}\ P_aY_{aI_aS} - P_aY_{aL_aA} - P_aY_{aL_aS} + P_fY_{fL_f}s - \lambda h_{L_f}s\ \ P_aY_{aI_aE}\ 0\quad -P_aY_{aL_aE} - h_{L_f} \right]$$

$$Y = \left[P_aY_{aI_aI_a}\ P_aY_{aI_aL_a}\quad -P_aY_{aL_aI_a}\ P_aG_{L_a} + P_fY_{fL_fL_f} - \lambda h_{L_fL_f} \right]$$

$$\left|Y\right| = P_aY_{aI_aI_a}\left\{P_aY_{aL_a} + P_fY_{fL_fL_f} - \lambda h_{L_fL_f}\right\} - \left(P_a\right)^2\left(Y_{aI_aL_a}\right)^2 \tag{48}$$

Comparative dynamic analysis for labour supply in forest extraction

$$\frac{dL_f}{dP_I} = \frac{-P_aY_{aL_aI_a}}{\left(P_a\right)^2\left\{Y_{aL_a}Y_{aI_aI_a} - \left(Y_{aI_aL_a}\right)^2\right\} + P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f} - \lambda h_{L_fL_f}\right\}} > 0 \tag{32}$$

$$\frac{dL_f}{dP_a} = \frac{-P_aY_{aI_aI_a}Y_{aL_a} + P_aY_{aL_aI_a}Y_{aI_a}}{\left(P_a\right)^2\left\{Y_{aL_a}Y_{aI_aI_a} - \left(Y_{aI_aL_a}\right)^2\right\} + P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f} - \lambda h_{L_fL_f}\right\}} < 0 \tag{33}$$

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$$\frac{dL_f}{dw}=\frac{-P_aY_{aI_aI_a}}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}<0 \quad (34)$$

$$\frac{dL_f}{dIN_s}=\frac{P_aY_{aI_aI_a}\left(-P_aY_{aL_a}+P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right)+(P_a)^2Y_{aL_aI_a}Y_{aI_a}s}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}<0 \quad (35)$$

$$\frac{dL_f}{dA_a}=\frac{-(P_a)^2\left(Y_{aI_aI_a}.Y_{aL_aA}\right)+(P_a)^2\left(Y_{aL_aI_a}.Y_{aI_aA}\right)}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}<0 \quad (37)$$

$$\frac{dL_f}{d\lambda}=\frac{-P_aY_{aI_aI_a}h_{L_f}}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}<0 \quad (38)$$

Comparative analysis for inputs used in agricultural activities

$$\frac{dI_a}{dP_I}=\frac{-P_aY_{aL_a}-P_fY_{fL_fL_f}+\lambda h_{L_{fL_f}}}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}<0 \quad (40)$$

$$\frac{dI_a}{dP_a}=\frac{Y_{aI_a}\left\{P_aY_{aL_a}+P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}+P_aY_{aI_aL_a}Y_{aL_a}}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}>0 \quad (41)$$

$$\frac{dI_a}{dw}=\frac{-P_aY_{aI_aL_a}}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}<0 \quad (42)$$

$$\frac{dI_a}{dIN_S}=\frac{P_aY_{aI_aIN_S}\left(P_aY_{aL_a}+P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right)+P_aY_{aI_aLa}\left(-P_aY_{aL_aS}+P_fY_{fL_f}s-\lambda h_{L_f}s\right)}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}>0 \quad (43)$$

$$\frac{dI_a}{dE_a}=\frac{P_aY_{aI_aE}\left(P_aY_{aL_a}+P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right)+(P_a)^2\left(Y_{aI_aL_a}.G_{I_aE}\right)}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}<0 \quad (45)$$

$$\frac{dI_a}{dA_a}=\frac{P_aY_{aI_aA}\left(P_aY_{aL_a}+P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right)+(P_a)^2\left(Y_{aI_aL_a}.Y_{aI_aA}\right)}{(P_a)^2\left\{Y_{aL_a}Y_{aI_aI_a}-\left(Y_{aI_aL_a}\right)^2\right\}+P_aY_{aI_aI_a}\left\{P_fY_{fL_fL_f}-\lambda h_{L_{fL_f}}\right\}}>0 \quad (46)$$

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ESM 2.

Changes in agricultural input prices

Differentiating Equation (21)–(24) w.r.t to agricultural input price we get

$$\partial\dot{\lambda}(t,\beta)=\rho\lambda(t,\beta)-\left[P_f h_{x_t}\left(x(t,\beta),\hat{L}_f(\lambda,\delta)\right)+\lambda(t,\beta)\left(F_{x_t}\left(x(t,\beta),\gamma,K\right)-h_{x_t}\left(x(t,\beta),\hat{L}_f(\lambda,\delta)\right)\right)\right]$$

$$\frac{\partial\dot{\lambda}(t,\beta)}{\partial P_i}=\dot{\lambda}_{P_i}=\rho\lambda_{P_i}-P_f h_{xx}\left(t,\beta^0\right)x_{P_i}-\lambda F_{xx}\left(t,\beta^0\right)x_{P_i}-F_{x_t}\left(t,\beta^0\right)\lambda_{P_i}+\lambda h_{xx}\left(t,\beta^0\right)x_{P_i}-h_{x_t}\left(t,\beta^0\right)\lambda_{P_i}$$

$$\dot{\lambda}_{P_i}=\rho\lambda_{P_i}+h_{x_t}\left(t,\beta^0\right)\lambda_{P_i}-F_{x_t}\left(t,\beta^0\right)\lambda_{P_i}-P_f h_{xx}\left(t,\beta^0\right)x_{P_i}-\lambda F_{xx}\left(t,\beta^0\right)x_{P_i}+\lambda h_{xx}\left(t,\beta^0\right)x_{P_i}$$

By rearranging the equation

$$\dot{\lambda}_{P_i}=\left\{\rho+h_{x_t}\left(t,\beta^0\right)-F_{x_t}\left(t,\beta^0\right)\right\}\lambda_{P_i}+\left\{\left(\lambda-P_f\right)h_{xx}\left(t,\beta^0\right)-\lambda F_{xx}\left(t,\beta^0\right)\right\}x_{P_i} \tag{49}$$

Now substituting the following in Equation (23) we get Equation (24).

$$\alpha_{11}\left(t,\beta^0\right)=\rho+h_{x_t}\left(t,\beta^0\right)-F_{x_t}\left(t,\beta^0\right)>0$$

$$\alpha_{12}\left(t,\beta^0\right)=\left(\lambda-P_f\right)h_{xx}\left(t,\beta^0\right)-\lambda F_{xx}\left(t,\beta^0\right)>0$$

$$\dot{x}(t,\beta)=F\left(x(t,\beta),\gamma,k\right)-h\left(x(t,\beta),\hat{L}_f(\lambda,\delta)\right)$$

$$\frac{\partial\dot{x}(t,\beta)}{\partial P_i}=X_{P_i}F_x\left(t,\beta^0\right)-X_{P_i}h_x\left(t,\beta^0\right)-\left\{h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{d\lambda}\left(t,\beta^0\right)\right\}\lambda_{P_i}-h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{dP_i}\left(t,\beta^0\right) \tag{50}$$

$$\dot{x}_{P_i}=\alpha_{21}\left(t,\beta^0\right)\lambda_{P_i}+\alpha_{22}\left(t,\beta^0\right)X_{P_i}-h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{dP_i}\left(t,\beta^0\right) \tag{51}$$

$$x_{P_i}(0)=0 \tag{52}$$

$$x_{P_i}\left(T^0\right)=0 \tag{53}$$

where:

$$\begin{aligned}\alpha_{21}\left(t,\beta^0\right)&=-\left\{h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{d\lambda}\left(t,\beta^0\right)\right\}>0\\ \alpha_{22}\left(t,\beta^0\right)&=\left\{F_x\left(t,\beta^0\right)-h_x\left(t,\beta^0\right)\right\}<0\end{aligned} \tag{54}$$

Defining the terms α_{11} and α_{12}, first, we need to explain the statement that $F_x < h_x$ which means that the steady-state forest stock growth rate is lower than the marginal harvesting impact on the stock. F_x is with a minus sign and to the right of the equilibrium value of forest stock while h_x is always positive. If the forest harvesting occurs to the right of the equilibrium value, in the inequality it automatically holds in the case of undisturbed forest. Although, if the equilibrium value of forest stock is lower than the maximum sustainable yield, then h_x indicates that the marginal harvest must be larger than the incremental growth in forest stock. Therefore, with positive h_x the extraction of forest resources increases with an increase in the equilibrium value of forest stock. Hence, this increase is greater than the further growth in the forest itself, so the statement of $F_x < h_x$ holds good.

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The solution of Equations (49) to (53) denoted by $\lambda_{P_i}(t, \beta^0)$ and $x_{P_i}(t, \beta^0)$ is continuous in $(t, \beta^0)\forall(t, \beta^0)\in(0, T^0)$, both these conditions describe the effect of agricultural input price changes on the steady-state growth of forest resources and their shadow price.

Summarising the above analysis in proposition form.

Proposition 2

(a) $x_{P_i}\left(t,\beta^0\right) \leq 0 \,\forall\, t \in \left(0, T^0\right)$

(b) $\lambda_{P_i}\left(t,\beta^0\right) \geq 0 \,\forall\, t \in \left(0, T^0\right)$

The qualitative properties of the Equation system (25)–(28) are described by the above proposition, which states that when the prices of inputs in agriculture increase, the cost of farming becomes expensive. Therefore, the representative household reallocates their labour time from agriculture to forest extraction, which indicates how an increase in agricultural input prices leads to lower forest stock at the optimal path.

Off-farm wages

$$\dot{\lambda}_{W_o} = \alpha_{11}\left(t,\beta^0\right)\lambda_{W_o} + \alpha_{12}\left(t,\beta^0\right)x_{W_o} \tag{55}$$

$$\dot{x}_{W_o} = \alpha_{21}\left(t,\beta^0\right)\lambda_{W_o} + \alpha_{22}\left(t,\beta^0\right)X_{W_o} - h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{dW_o}\left(t,\beta^0\right) \tag{56}$$

$$x_{W_o}\left(0\right) = 0 \tag{57}$$

$$x_{W_o}\left(T^0\right) = 0 \tag{58}$$

Proposition 3

(a) $x_{W_o}\left(t,\beta^0\right) \geq 0 \,\forall\, t \in \left(0, T^0\right)$

(b) $\lambda_{W_o}\left(t,\beta^0\right) \leq 0 \,\forall\, t \in \left(0, T^0\right)$

Proposition 3 shows that an increase in off-farm wages induces a shift of labour from forest extraction and non-agricultural activities. An increase in off-farm wages leads to the higher equilibrium stock of forest associated with the lower shadow price of forest.

Social Security (Income Support Program)

$$\dot{\lambda}_{S_c} = \alpha_{11}\left(t,\beta^0\right)\lambda_{S_c} + \alpha_{12}\left(t,\beta^0\right)x_{S_c} \tag{59}$$

$$\dot{x}_{S_c} = \alpha_{21}\left(t,\beta^0\right)\lambda_{S_c} + \alpha_{22}\left(t,\beta^0\right)X_{S_c} - h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{dS_c}\left(t,\beta^0\right) \tag{60}$$

$$x_{S_c}\left(0\right) = 0 \tag{61}$$

$$x_{S_c}\left(T^0\right) = 0 \tag{62}$$

Proposition 4

(c) $x_{S_c}\left(t,\beta^0\right) \geq 0 \,\forall\, t \in \left(0, T^0\right)$

(d) $\lambda_{S_c}\left(t,\beta^0\right) \leq 0 \,\forall\, t \in \left(0, T^0\right)$

The properties of the above system of the equations show that government social security or income support program leads to lower forest dependency for income. Which further raises the equilibrium value of forest stock and lowers the shadow value of the forest.

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Change in agricultural product prices

$$\dot{\lambda}_{P_a} = \alpha_{11}\left(t,\beta^0\right)\lambda_{P_a} + \alpha_{12}\left(t,\beta^0\right)x_{P_a} \tag{63}$$

$$\dot{x}_{P_a} = \alpha_{21}\left(t,\beta^0\right)\lambda_{P_a} + \alpha_{22}\left(t,\beta^0\right)X_{P_a} - h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{dP_a}\left(t,\beta^0\right) \tag{64}$$

$$x_{P_a}\left(0\right) = 0 \tag{65}$$

$$x_{P_a}\left(T^0\right) = 0 \tag{66}$$

The above system of differential equations is summarized below.

Proposition 5

(e) $x_{P_a}\left(t,\beta^0\right) \geq 0 \,\forall\, t \in \left(0,T^0\right)$

(f) $\lambda_{P_a}\left(t,\beta^0\right) \leq 0 \,\forall\, t \in \left(0,T^0\right)$

The qualitative properties of the above system of the equations show that when the price of agricultural products increases, it leads to a labour shift from forest collection to farming. These shifts in labour supply further lead to the higher forest stock at the steady-state growth path. Therefore, agricultural product prices create an incentive for a rural household to engage in farming due to a higher return.

Change in forest product prices

$$\dot{\lambda}_{P_f} = \alpha_{11}\left(t,\beta^0\right)\lambda_{P_f} + \alpha_{12}\left(t,\beta^0\right)x_{P_f} \tag{67}$$

$$\dot{x}_{P_f} = \alpha_{21}\left(t,\beta^0\right)\lambda_{P_f} + \alpha_{22}\left(t,\beta^0\right)X_{P_f} - h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{dP_f}\left(t,\beta^0\right) \tag{68}$$

$$x_{P_f}\left(0\right) = 0 \tag{69}$$

$$x_{P_f}\left(T^0\right) = 0 \tag{70}$$

Proposition 6

(g) $x_{P_f}\left(t,\beta^0\right) \leq 0 \,\forall\, t \in \left(0,T^0\right)$

(h) $\lambda_{P_f}\left(t,\beta^0\right) \geq 0 \,\forall\, t \in \left(0,T^0\right)$

The above proposition indicates that when forest product prices increase, they attract rural households to engage in forest collection. Higher forest prices increase the opportunity cost of farming and other off-farm activities. So household labour supply in agricultural activities decreases and it increases in forest extraction. This phenomenon leads to lower forest resources at the steady-state growth path.

Change in the area under agricultural products

$$\dot{\lambda}_{A_a} = \alpha_{11}\left(t,\beta^0\right)\lambda_{A_a} + \alpha_{12}\left(t,\beta^0\right)x_{A_a} \tag{71}$$

$$\dot{x}_{A_a} = \alpha_{21}\left(t,\beta^0\right)\lambda_{A_a} + \alpha_{22}\left(t,\beta^0\right)X_{A_a} - h_{L_f}\left(t,\beta^0\right)\frac{d\hat{L}_f}{dA_a}\left(t,\beta^0\right) \tag{72}$$

$$x_{A_a}\left(0\right) = 0 \tag{73}$$

$$x_{A_a}\left(T^0\right) = 0 \tag{74}$$

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Proposition 7

(i) $x_{A_a}\left(t,\beta^0\right) \quad 0\,\forall\, t\in\left(0,T^0\right)$

(j) $\lambda_{A_a}\left(t,\beta^0\right)\leq 0\,\forall\, t\in\left(0,T^0\right)$

The qualitative properties of the above system of equations show that when the area under agricultural production increases, it has both negative and positive effects on the equilibrium value of forest stock. When the forest area is defined by protected forest and strictly non-convertible to farming, then the expansion in agriculture leads to the higher equilibrium value of forest stock. Because as the area under agriculture increases, it requires more inputs and labour time, secondly agriculture in a rural area is too labour-intensive, so households reduce their labour time in forest extraction and increase their time in farming. The negative effect of agricultural expansion mainly occurs due to the illegal acquisition of forest areas. In this case, the lower equilibrium value of forest stock occurs due to agricultural expansion.