https://doi.org/10.17221/44/2026-JFS

Optimisation of conversion of structurally homogeneous forest stands into uneven-aged forest structures using a multi-period Liocourt model

Jan Kašpar, Zohreh Mohammadi, Meryem Tahri*

Faculty of Forestry and Wood Sciences, Czech University of Life Sciences Prague, Prague, Czech Republic

**Corresponding author: kasparj@fld.czu.cz*

Citation: Kašpar J., Mohammadi Z., Tahri M. (2026): Optimisation of conversion of structurally homogeneous forest stands into uneven-aged forest structures using a multi-period Liocourt model. J. For. Sci., 72: 340–355.

Abstract: Uneven-aged forest management and Continuous Cover Forestry (CCF) are increasingly discussed as adaptive silvicultural approaches capable of improving forest resilience under climate change, increasing disturbance frequency, and growing uncertainty in forest ecosystems. However, the conversion of structurally homogeneous stands into uneven-aged structures represents a complex optimisation problem involving economic performance, structural stability, regeneration continuity, and long-term sustainability. This study presents a multi-period optimisation framework based on size-class dynamics, transition matrices, and Liocourt-based structural regulation. The model is formulated as a quadratic optimisation problem that maximises net present value while simultaneously minimising deviations from a target uneven-aged diameter distribution. A hypothetical case study was developed to analyse the conversion of structurally homogeneous stands toward uneven-aged structures regulated by Liocourt distributions. The optimisation experiments included extensive sensitivity analyses involving combinations of q-ratio, target basal area, recruitment intensity, growth dynamics, and mortality parameters. Results demonstrate that recruitment and lower diameter class dynamics represent the dominant drivers of successful structural conversion, while harvesting alone cannot compensate for insufficient regeneration inflow. The analysis further confirms that Liocourt distributions should not be interpreted as universal biological equilibria but rather as regulatory and optimisation targets dependent on ecological, silvicultural, and economic conditions.

Keywords: BDq method; Continuous Cover Forestry; forest optimisation; forest planning; harvest scheduling; Liocourt law; quadratic programming; size-class model; transition matrix; uneven-aged forest management

European forest management has increasingly shifted from narrowly production-oriented systems toward multifunctional and resilience-oriented silviculture. This transition has been accelerated by climate change, disturbance outbreaks, biodiversity decline, and increasing societal demand for ecosystem services. Within this context, Continuous Cover Forestry (CCF) and close-to-nature silviculture have gained renewed attention as potential alternatives or complements to conventional even-aged management (O'Hara 2023).

Supported by the Ministry of Agriculture, National Agency for Agricultural Research (Project No. QK21010354 'Progressive methods of forest management planning to support sustainable forest management').

https://doi.org/10.17221/44/2026-JFS

CCF is based on the long-term maintenance of forest cover, avoidance of large clear-cuts, continuous regeneration, and structural diversification. In Central European forestry, this logic is closely related to selection systems, irregular shelterwood systems, and mixed-species close-to-nature management. Brang et al. (2014) argued that many principles of close-to-nature silviculture can contribute to climate adaptation in temperate European forests, particularly by increasing structural diversity, supporting tree species richness, improving resistance of individual trees to abiotic and biotic stress, and reducing risks associated with high growing stocks. Recent work has further emphasised the role of optimisation and adaptive decision-making in CCF systems (Hyytiäinen, Haight 2012).

Uneven-aged forest management represents one of the most structurally demanding forms of CCF. It is based on repeated partial harvesting and continuous regeneration, with the stand regulated not by age-class area but by diameter-class structure. The objective is not to regenerate an entire stand at once, but to maintain a permanent flow of trees across diameter classes, from regeneration and ingrowth to merchantable dimensions. Consequently, uneven-aged management is a dynamic regulation problem in which harvesting, growth, mortality, and recruitment interact over time.

The defining feature of uneven-aged management is the coexistence of trees of different sizes, ages, and canopy positions within the same stand or management unit. This creates a fundamentally different planning problem from even-aged forestry. In even-aged systems, management decisions are typically organised around stand age, rotation length, thinning schedules, and final harvest. In uneven-aged systems, the relevant state variable is the diameter or size distribution of trees.

Stocking regulation in uneven-aged stands has historically relied on the assumption that sustainable selection forests can be described by balanced diameter distributions. Such distributions are usually characterised by many small trees and progressively fewer larger trees. This form is commonly described as an inverse-J or negative exponential distribution. However, empirical studies have shown that real managed uneven-aged stands often deviate substantially from idealised diameter structures. Schwartz et al. (2005), for example, found that uneven-aged management in northern hardwoods produced different diameter distributions and species compositions depending on ownership and marking practice, indicating that structural regulation is not automatic but depends on explicit management rules. More recent empirical work confirms that BDq-based regulation can produce highly variable outcomes depending on site conditions and marking intensity (Brzeziecki, Neroj 2024).

This is important for forest planning because structural diversity alone does not guarantee sustainability. A stand may contain several diameter classes but still lack sufficient recruitment into lower classes, or may accumulate surplus trees in medium classes while failing to produce enough future crop trees. Therefore, the conversion of structurally homogeneous stands into uneven-aged systems must be understood as a controlled transition of the entire diameter distribution, not merely as repeated partial harvesting.

The classical theoretical reference for uneven-aged structural regulation is Liocourt's law. In its simplified modern interpretation, the number of trees in successive diameter classes follows a geometric progression. The ratio between the number of trees in one diameter class and the next larger class is known as the *q*-ratio. A constant *q*-ratio generates a smooth decreasing diameter distribution and has been widely used as a practical guide for selection forestry. Contemporary analyses show that Liocourt-type structures remain useful but require ecological calibration (Brzeziecki, Neroj 2024).

Kerr (2014) revisited the original work of de Liocourt and emphasised that the historical interpretation of Liocourt's results has often been simplified. De Liocourt's observations were based on silver fir stands managed under specific ecological and silvicultural conditions, and the resulting diameter structures should not be interpreted as universal biological equilibria. Picard and Gasparotto (2016) similarly argued that Liocourt's law is better understood as a theoretical or statistical representation of diameter distributions rather than a general law that all uneven-aged forests necessarily follow.

The practical application of Liocourt-type regulation is often expressed through the BDq method. The method defines a target stand structure using three main parameters: target basal area (B), maximum diameter (D), and q-ratio (q). This approach provides a simple and operationally useful framework for marking and stocking control. However,

https://doi.org/10.17221/44/2026-JFS

its simplicity also creates risks. If *B*, *D*, and *q* are selected without considering site productivity, species composition, regeneration capacity, mortality, and economic conditions, the resulting target structure may be biologically unrealistic or economically inefficient (Brzeziecki, Neroj 2024).

Therefore, in this study, Liocourt distributions are not treated as natural equilibria. They are treated as regulatory targets used to guide optimisation. This distinction is central: the model does not assume that the forest will automatically converge to a Liocourt curve. Instead, it evaluates whether, and under which biological and economic assumptions, such convergence can be achieved through a sequence of harvest decisions.

Because uneven-aged management is organised around diameter structure, size-class and transition matrix models are particularly suitable for its analysis. Buongiorno and Michie (1980) developed a seminal matrix model of uneven-aged forest management in which tree numbers are projected across diameter classes using transition probabilities. This type of model provides a computationally efficient representation of growth, mortality, and ingrowth while retaining the structural detail required for selection forestry.

Matrix models offer an intermediate level of complexity between whole-stand models and individual-tree models. Whole-stand models are simple and robust but insufficiently sensitive to diameter structure. Individual-tree models provide detailed biological realism but require extensive data and calibration. Transition matrix models, by contrast, can represent the main structural processes needed for optimisation: survival within a diameter class, transition to the next class, mortality, and recruitment into the smallest class.

Several studies have used transition matrices or related size-class models to analyse uneven-aged management. Hao et al. (2005), for example, developed a transition matrix growth and ingrowth model for uneven-aged mixed-species forests, showing the importance of species-specific upgrowth and ingrowth dynamics. Virgilietti and Buongiorno (1997) emphasised that matrix models are flexible enough to represent even-aged, uneven-aged, monospecific, and mixed-species stands while remaining computationally tractable for management applications.

For the present study, the transition matrix framework is particularly appropriate because conversion from homogeneous to uneven-aged structures requires repeated projection of the stand distribution after harvest. Each management cycle consists of three linked processes: removal of selected trees, growth and survival of the residual stand, and recruitment into the smallest diameter classes.

A central difficulty in modelling uneven-aged stands is the uncertainty of recruitment and mortality. In even-aged management, regeneration is usually concentrated in a planned regeneratio phase. In uneven-aged management, regeneration and ingrowth must occur continuously. If recruitment into lower diameter classes is insufficient, the diameter distribution gradually becomes unstable regardless of how carefully larger trees are harvested.

Recruitment is influenced by canopy structure, light conditions, seed availability, browsing pressure, soil conditions, species shade tolerance, and disturbance history. It is therefore difficult to represent using a single constant parameter. Mortality is similarly variable and may depend on competition, diameter class, species, stand density, drought stress, pathogens, and stochastic disturbance events.

This creates a major challenge for optimisation. A harvest schedule that appears optimal under one recruitment assumption may fail under another. The lower diameter classes are especially important because they represent the future structural and economic capacity of the stand. If the inflow into these classes is underestimated or overestimated, long-term projections of structural stability may be strongly biased.

For this reason, recruitment and mortality are treated in this study not only as biological parameters but also as key uncertainty drivers. The scenario analysis explicitly tests how different recruitment, growth, and mortality assumptions affect structural convergence toward Liocourt-type target distributions.

Uneven-aged forest management has long been recognised as a dynamic optimisation problem. Classical approaches include dynamic programming, optimal control, matrix-based optimisation, and, more recently, nonlinear programming, mixed-integer programming, and heuristic optimisation.

Haight (1985) compared dynamic and static economic models of uneven-aged stand management and showed that dynamic formulations are essential when harvest decisions affect future stand structure and economic returns. Haight et al. (1985) further

https://doi.org/10.17221/44/2026-JFS

developed optimisation approaches for sequences of diameter distributions and selection harvests. These studies established the conceptual basis for treating uneven-aged management as an intertemporal control problem.

Dynamic programming provides a theoretically powerful framework, but its practical application is often limited by dimensionality. The number of possible states increases rapidly with the number of diameter classes, species, and management constraints. For this reason, matrix-based optimisation and nonlinear programming provide attractive alternatives for operational models.

López-Torres et al. (2016) formulated uneven-aged *Pinus nigra* management as a discrete optimal control problem maximising net present value between stable stand structures. Their work is particularly relevant because it links financial optimisation with matrix-based stand dynamics. Similarly, Pukkala et al. (2010) optimised steady-state structures and management of uneven-sized pine and spruce stands, demonstrating that optimal structures depend strongly on economic assumptions and species-specific growth conditions.

At broader spatial scales, forest harvest scheduling has often been formulated using mixed-integer programming. Constantino et al. (2008) developed a mixed-integer programming model for harvest scheduling subject to maximum area restrictions. Although such models were mainly developed for even-aged or spatial harvest scheduling problems, they provide useful methodological foundations for extending uneven-aged optimisation to multiple stands and landscape-level planning.

Despite substantial literature on uneven-aged management, several gaps remain. First, Liocourt and BDq structures are often used as practical targets, but their role in dynamic optimisation is not always clearly distinguished from biological equilibrium assumptions. Second, many studies optimise steady-state conditions, while practical conversion from structurally homogeneous stands requires a transition path. Third, recruitment and mortality are often treated as fixed parameters, although they may dominate long-term feasibility. Fourth, there is a need for transparent models that combine economic optimisation, structural regulation, and stand dynamics in a form that can be implemented using modern quadratic programming solvers.

This study addresses these gaps by formulating a multi-period Liocourt-based optimisation model for the conversion of structurally homogeneous forest stands into uneven-aged structures. The model represents the stand as a vector of diameter-class frequencies, applies a transition matrix to simulate growth and mortality, includes recruitment into the smallest class, and optimises harvest decisions over multiple management cycles.

The objective function combines discounted economic return with a quadratic penalty for deviations from the target Liocourt distribution. This formulation makes it possible to analyse explicit trade-offs between short-term harvest revenue and long-term structural convergence. The model also enables scenario analysis of the q-ratio, target basal area, recruitment, growth, and mortality.

The main contribution of the study is therefore not the claim that Liocourt distributions represent universal biological equilibria, but the development of a formal optimisation framework in which Liocourt-type structures are treated as adjustable regulatory targets. This provides a transparent basis for testing the feasibility, robustness, and economic implications of uneven-aged conversion strategies.

MATERIAL AND METHODS

Conceptual framework

The proposed framework was formulated as a multi-period discrete-time optimisation problem. The stand state was represented by vectors of tree frequencies in diameter classes. Structural development was modelled using transition matrices describing growth and mortality processes between management cycles.

The framework combines size-class stand dynamics, harvest optimisation, structural regulation, economic evaluation, and sensitivity analysis.

The optimisation horizon consisted of repeated management cycles representing periodic selective harvesting.

State-space representation

Indices. The following indices were used:

– t – management cycle;
– i – diameter class;
– T – planning horizon;
– n – number of diameter classes.

State variables. The stand structure at time t was represented by $N_{i,t}$, denoting the number of trees in diameter class i at time t.

https://doi.org/10.17221/44/2026-JFS

Decision variables. Harvest decisions were represented by $H_{i,t}$, denoting the number of harvested trees in class *i* during cycle *t*.

The remaining stand after harvest was calculated according to Equation (1):

$$R_{i,t} = N_{i,t} - H_{i,t} \quad (1)$$

where:
$N_{i,t}$ – number of trees in diameter class *i* at time *t*;
$H_{i,t}$ – number of harvested trees in class *i* during cycle *t*.

Dynamic stand model

Stand development between management cycles was described using a transition matrix framework, see Equation (2):

$$N_{t+1} = A(N_t - H_t) + B_t \quad (2)$$

where:
A – transition matrix describing growth and mortality;
B_t – the recruitment vector.

The transition matrix contains probabilities or deterministic coefficients describing transitions between neighbouring diameter classes.

Transition matrix structure

The hypothetical case study used the following simplified transition matrix, as shown by Equation (3):

$$A = \begin{bmatrix} 0.82 & 0 & 0 & 0 & 0 \\ 0.15 & 0.80 & 0 & 0 & 0 \\ 0 & 0.16 & 0.79 & 0 & 0 \\ 0 & 0 & 0.17 & 0.78 & 0 \\ 0 & 0 & 0 & 0.18 & 0.77 \end{bmatrix} \quad (3)$$

The recruitment vector was defined by Equation (4):

$$B_t = [b_t, 0,0,0,0]^T \quad (4)$$

where:
B_t – recruitment into the smallest diameter class;
T – planning horizon.

Target stand structure

The target uneven-aged structure was defined using Liocourt's law, see Equation (5):

$$N_i^* = N_1^* q^{-(i-1)} \quad (5)$$

where:
N_i^* – target number of trees in class *i*;
q – Liocourt ratio.

Calibration of the initial target class was based on target basal area, see Equation (6):

$$G^* = \sum_i g_i N_i^* \quad (6)$$

where:
G^* – target basal area;
g_i – basal area per tree in class *i*.

Objective function

The optimisation objective combined economic optimisation and structural regulation.

The objective function was formulated as Equation (7):

$$max \sum_{t=1}^{T} \frac{R_t(H_t) - C_t}{(1+r)^t} - \lambda \sum_{t=1}^{T} \sum_{i=1}^{n} \left(N_{i,t} - N_i^*\right)^2 \quad (7)$$

where:
$R_t(H_t)$ – economic return from harvesting;
C_t – fixed management costs;
r – the discount rate;
λ – the structural penalty parameter.

The first term maximises discounted economic return, while the second penalises deviations from the target uneven-aged structure.

Constraints

Harvest availability – Equation (8):

$$0 \le H_{i,t} \le N_{i,t} \quad (8)$$

Minimum basal area – Equation (9):

$$\sum_i g_i \left(N_{i,t} - H_{i,t}\right) \ge G_{\min} \quad (9)$$

Maximum harvest intensity – Equation (10):

$$H_{i,t} \le \alpha N_{i,t} \quad (10)$$

where:
α – maximum allowable harvest intensity.

https://doi.org/10.17221/44/2026-JFS

Diameter classes below target structure were protected. Protection of deficit classes – Equation (11):

$$H_{i,t} = 0 \text{ if } N_{i,t} < N_i^* \quad (11)$$

Recruitment constraint – Equation (12):

$$B_t \geq B_{\min} \quad (12)$$

The recruitment constraint was introduced because regeneration continuity represents one of the critical determinants of long-term uneven-aged stability.

Scenario analysis

A comprehensive sensitivity analysis was conducted using combinations of *q*-ratio, target basal area, recruitment intensity, mortality, growth coefficients, and discount rates. A total of 432 optimisation scenarios were simulated (Table 1).

The scenarios were designed to evaluate structural convergence, economic performance, recruitment sensitivity, robustness of target structures, and long-term feasibility of uneven-aged conversion.

Growth coefficients controlled transitions among diameter classes between management cycles, mortality parameters represented losses occurring within each cycle, and recruitment values determined the number of new trees entering the smallest diameter class. Economic assumptions included harvest revenues, fixed management costs, and a constant discount rate applied throughout the planning horizon. The selected parameter values were intended to evaluate model sensitivity and structural behaviour rather than represent a specific forest type or management system.

Numerical implementation

The optimisation model was implemented as a quadratic programming problem compatible with Gurobi Optimizer (Version 13.0.2, 2026).

Table 1. Sets of designed parameters

Parameter	Values
q	1.15, 1.25, 1.35, 1.45
B	14, 16, 18, 20
Growth	slow, medium, fast
Mortality	low, medium, high
Recruitment	low, medium, high

q – *q*-ratio; *B* – target basal area

The optimisation procedure was performed iteratively for each management cycle. For every cycle, the solver determined harvest intensities, minimising structural deviation while maximising economic return.

RESULTS

Hypothetical stand conversion. The model was tested on a hypothetical structurally homogeneous stand characterised by a substantial surplus in medium diameter classes and a deficit in lower diameter classes. The initial stand structure represented a typical transformed production forest with insufficient regeneration inflow and structural imbalance.

The initial diameter distribution consisted of 11 diameter classes ranging from 12 cm to 52 cm diameter at breast height (*DBH*) (Table 2).

The target structure was defined using a Liocourt distribution and calibrated according to the target basal area. The optimisation was implemented as a multi-period quadratic programming problem using repeated management cycles.

The model systematically reduced surplus trees in medium and upper diameter classes while simultaneously protecting deficit lower classes from harvesting. The resulting trajectories gradually converged toward the target Liocourt distribution.

The optimisation revealed that the deficit lower diameter classes could not be restored through harvesting strategies alone. Instead, long-term convergence strongly depended on recruitment and transition dynamics.

Table 2. Hypothetical initial *DBH* structure

Diameter class	Mean *DBH* (cm)	Initial tree number (trees·ha^{-1})
1	12	80
2	16	110
3	20	140
4	24	130
5	28	100
6	32	70
7	36	45
8	40	30
9	44	20
10	48	12
11	52	6

DBH – diameter at breast height

https://doi.org/10.17221/44/2026-JFS

The model was tested on a hypothetical, structurally homogeneous stand characterised by surplus medium and upper diameter classes, deficit lower classes, and insufficient regeneration inflow.

Structural convergence was accompanied by a decline in harvest intensity during the initial conversion phases and by the stabilisation of harvest levels after partial structural equilibration.

Numerical solution of the hypothetical quadratic programming (QP) model. A simplified numerical optimisation experiment was conducted using five diameter classes and three management cycles. The initial and target diameter distribution is presented in Table 3. The optimisation produced the harvest intensities presented in Table 4.

The model intentionally avoided harvesting deficit lower classes and concentrated harvesting pressure into structurally surplus medium and upper classes. Harvest and basal area dynamics are presented in Table 5. The terminal state after three management cycles approached the target Liocourt structure, as presented in Table 6.

Table 3. Initial and target diameter distribution for numerical optimisation experiment

Diameter class	*DBH* (cm)	Initial state (trees·ha^{-1})	Target Liocourt state (trees·ha^{-1})
1	12	80	182.824
2	16	150	140.634
3	20	130	108.180
4	24	90	83.215
5	28	35	64.012

DBH – diameter at breast height

Table 4. Resulted harvest intensities

Cycle	Class 1	Class 2	Class 3	Class 4	Class 5
0	0.000	0.000	21.312	9.066	3.405
1	0.000	0.000	8.256	6.147	19.266
2	0.000	0.000	0.000	11.567	18.955

The results demonstrate that the optimisation effectively reduced structural surplus in medium classes while gradually approaching the target diameter distribution. However, substantial deficits in lower classes persisted despite the absence of harvesting in these classes.

This finding confirms that harvesting cannot compensate for insufficient regeneration inflow and that successful conversion toward uneven-aged structures fundamentally depends on long-term recruitment dynamics.

Influence of recruitment dynamics. Recruitment intensity emerged as the dominant determinant of successful uneven-aged conversion.

Scenarios with insufficient recruitment failed to maintain the target lower diameter classes despite reduced harvesting pressure. In contrast, high recruitment scenarios achieved stable structural convergence even under relatively intensive harvesting.

Recruitment intensity emerged as the dominant driver of successful uneven-aged conversion (Figure 1). Average terminal Root Mean Square Error (*RMSE*) decreased strongly with increasing recruitment intensity, while low-recruitment scenarios produced highly unstable terminal structures regardless of harvesting strategy.

Influence of *q*-ratio. The *q*-ratio strongly influenced optimal structural trajectories (Figure 2). Higher *q*-ratios increased pressure on upper diameter class harvesting and produced steeper target diameter distributions. Low *q*-ratios generated flatter distributions with higher proportions of large trees.

Extremely high *q*-ratios often resulted in structurally unstable solutions sensitive to small recruitment fluctuations. Moderate *q*-ratios produced the most robust long-term solutions.

The robustness landscape demonstrates that structurally feasible conversion trajectories are concentrated in scenarios with lower target basal area and moderate *q*-ratios. Higher *q*-values substantially increase terminal structural instabil-

Table 5. Harvest and basal area dynamics

Cycle	Harvested trees (trees·ha^{-1})	Harvest volume (m^3·ha^{-1})	Basal area before harvest (m^2·ha^{-1})	Basal area after harvest (m^2·ha^{-1})
0	33.783	13.480	14.231	12.942
1	33.669	20.000	14.199	12.476
2	30.522	20.000	13.690	12.000

https://doi.org/10.17221/44/2026-JFS

Table 6. Terminal state after three management cycles

Diameter class	Target state	Terminal state
1	182.824	97.012
2	140.634	105.391
3	108.180	107.283
4	83.215	79.479
5	64.012	48.690

ity because they require stronger recruitment into lower diameter classes. This figure (Figure 2) directly supports the discussion that Liocourt distributions should be treated as optimisation targets, the feasibility of which depends on biological recruitment dynamics.

Mortality and growth sensitivity. Mortality strongly affected long-term structural stability. Elevated mortality in lower and medium diameter classes substantially reduced the feasibility of maintaining target structures. Mortality effects became particularly important under low recruitment conditions.

Growth intensity influenced transition speed between diameter classes and therefore affected the temporal stability of structural regulation.

Economic–structural trade-offs. Increasing the structural penalty parameter λ strengthened structural stabilisation while reducing short-term economic returns.

Scenarios with low λ prioritised harvest revenues and frequently produced structurally unstable outcomes. Conversely, very high λ values generated conservative harvest strategies associated with lower net present value. Intermediate values of λ provided the most balanced solutions.

Theoretical analysis of the objective function. For a better interpretation of the behaviour of the optimisation model, an analytical investigation of the objective function of the multiperiod discrete model was carried out. The aim was to examine the local properties of the optimisation problem, the economic interpretation of the gradients, and the conditions for the existence of a unique optimum.

The original objective function was formulated as Equation (13):

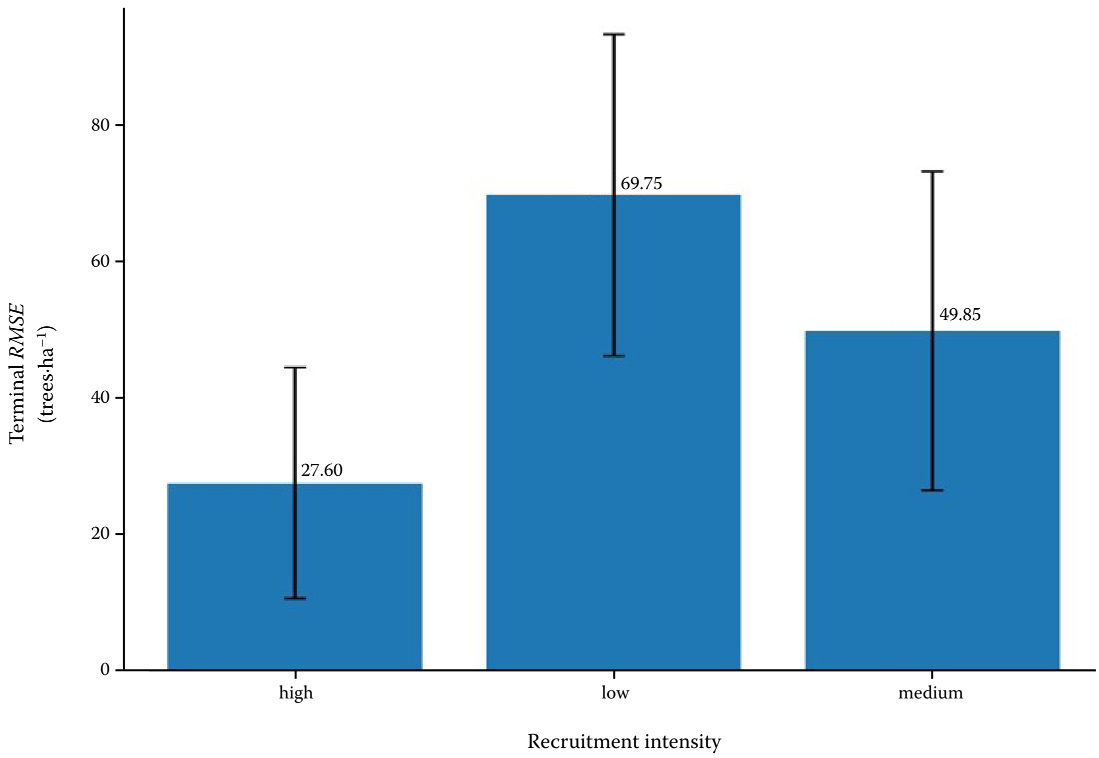

Figure 1. Effect of recruitment intensity on the terminal Root Mean Square Error (*RMSE*) of stand structure

Bars – mean *RMSE* values across all simulated scenarios; error bars indicate ±1 standard deviation; lower *RMSE* values indicate stronger convergence toward the target Liocourt diameter distribution; numerical labels above the bars show mean *RMSE* values (trees·ha^{-1})

https://doi.org/10.17221/44/2026-JFS

$$\max Z = \sum_{t=0}^{T-1} \frac{\sum_k \pi_k H_{k,t} - F_t}{(1+i)^t} - \lambda \sum_{t=0}^{T-1} \sum_k \left(X_{k,t} - N_k^*\right)^2 \tag{13}$$

where:

$H_{k,t}$ – harvesting intensity;

$X_{k,t} = N_{k,t} - H_{k,t}$ – the residual stand state after intervention;

N_k^* – target Liocourt distribution;

π_k – the net economic return from harvesting;

λ – the structural penalty parameter.

The model therefore combines economic maximisation with a regulatory effort to guide the stand toward the target uneven-aged structure.

Partial derivative with respect to harvesting. The partial derivative of the objective function with respect to the decision variable $H_{k,t}$ is Equation (14):

$$\frac{\partial Z}{\partial H_{k,t}} = \frac{\pi_k}{(1+i)^t} + 2\lambda\left(X_{k,t} - N_k^*\right) \tag{14}$$

Substituting the relationship $X_{k,t} = N_{k,t} - H_{k,t}$, the derivative can be rewritten as Equation (15):

$$\frac{\partial Z}{\partial H_{k,t}} = \frac{\pi_k}{(1+i)^t} + 2\lambda\left(N_{k,t} - H_{k,t} - N_k^*\right) \tag{15}$$

The economic interpretation of the derivative shows that optimal harvesting results from two opposing forces: the economic pressure to maximise immediate harvesting revenues and the structural pressure to minimise deviations from the target Liocourt distribution.

If a diameter class contains an excess number of trees relative to the target state, the structural term promotes more intensive harvesting. Conversely, deficit classes are protected from intervention by the structural penalty. This mechanism corresponds to the results of the numerical simulations, in which the model systematically preserved lower deficit classes without harvesting.

Interior optimum. The first-order condition – Equation (16):

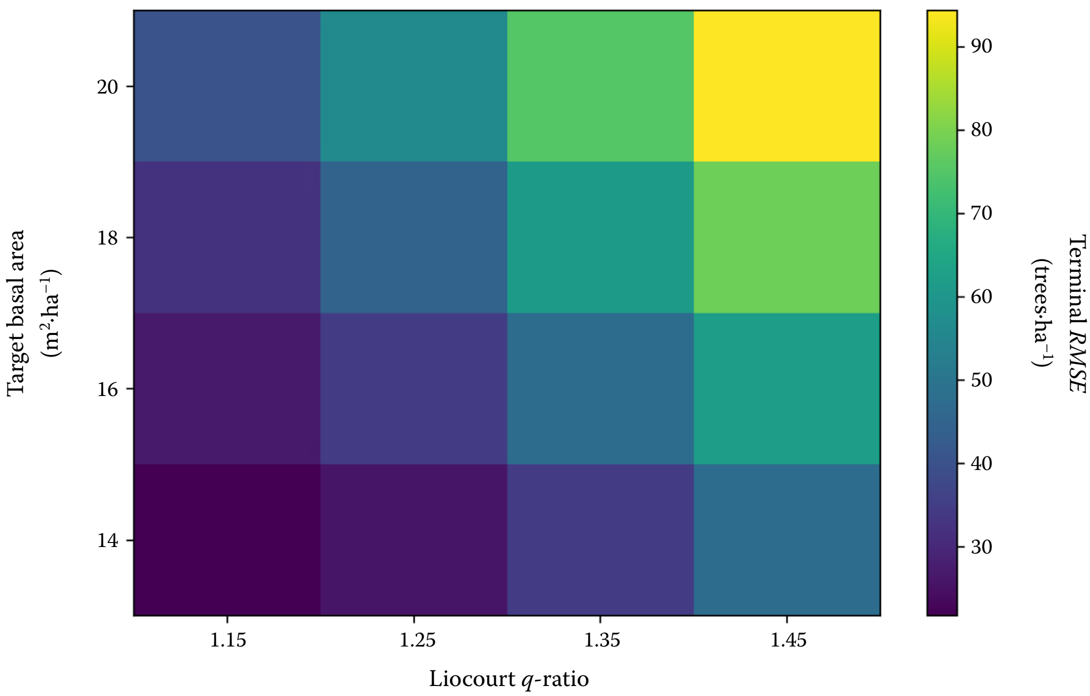

Figure 2. Average terminal Root Mean Square Error (*RMSE*) across 432 optimisation scenarios as a function of target basal area (m^2·ha^{-1}) and Liocourt *q*-ratio

Lower *RMSE* values indicate stronger structural convergence toward the target uneven-aged diameter distribution; the colour scale represents average terminal *RMSE* (trees·ha^{-1})

https://doi.org/10.17221/44/2026-JFS

$$\frac{\partial Z}{\partial H_{k,t}} = 0 \tag{16}$$

This leads to the analytical expression for optimal harvesting intensity – Equation (17):

$$H_{k,t}^{opt} = N_{k,t} - N_k^* + \frac{\pi_k}{2\lambda(1+i)^t} \tag{17}$$

The optimal harvesting intensity therefore consists of two components: the structural surplus relative to the target distribution, an economic correction dependent on assortment profitability and discounting.

An important result is that the economic effect of harvesting is discounted over time. Consequently, in later management cycles, the model gradually prioritises structural stabilisation of the stand over short-term economic maximisation.

Concavity of the objective function. The second derivative with respect to $H_{k,t}$ is:

$$\frac{\partial^2 Z}{\partial H_{k,t}^2} = -2\lambda \quad \text{for } \lambda > 0 \tag{18}$$

The second derivative is negative, implying that the objective function is concave with respect to the harvesting decision variables.

This result is highly significant from an optimisation perspective because it: guarantees the existence of a unique local maximum, simultaneously implies a global optimum within the continuous part of the problem and enables efficient solution using convex quadratic programming. This explains the stable numerical behaviour of the optimisation model even under a large number of scenarios.

Sensitivity to the structural regulation parameter. The partial derivative with respect to parameter λ is Equation (19):

$$\frac{\partial Z}{\partial \lambda} = -\sum_t \sum_k (X_{k,t} - N_k^*)^2 \leq 0 \tag{19}$$

Increasing the parameter λ therefore always decreases the objective function value because it strengthens the penalty for structural deviations. Practically, this represents a transition from economically oriented forest management toward a structurally conservative management regime.

Low values of λ lead to more aggressive harvesting of economically profitable diameter classes, whereas high values stabilise stand distribution and reduce structural deviations.

Sensitivity to the *q*-ratio. Since the target structure depends on the *q*-ratio, Equation (20) applies:

$$N_k^* = N_1^* q^{-(k-1)} \tag{20}$$

The sensitivity of the objective function with respect to the parameter can also be derived from Equation (21):

$$\frac{\partial Z}{\partial q} = -2\lambda \sum_t \sum_k (X_{k,t} - N_k^*) \frac{\partial N_k^*}{\partial q} \tag{21}$$

The analysis indicates that higher *q*-values lead to a steeper decline in the abundance of larger diameter classes and therefore to more intensive harvesting interventions in medium and upper diameter classes. Conversely, lower *q*-values support a flatter distribution and greater stand stock stability.

The results of the scenario analysis suggest that excessively high *q*-values may lead to unstable stand structures and increased model sensitivity to recruitment dynamics.

Interpretation from the perspective of forest management. The theoretical analysis demonstrates that the optimisation model does not function merely as an economic harvest scheduling tool, but rather as a regulatory mechanism governing forest structural dynamics. Harvesting therefore serves not only a production function, but also acts as a tool for directing the stand toward the target distribution.

From a mathematical perspective, the model combines a discrete dynamic system, convex quadratic programming, control theory and economic optimisation.

This combination makes it possible to interpret stand transformation as a problem of determining the optimal trajectory of forest development over time, rather than merely as a sequence of isolated harvesting decisions.

DISCUSSION

The results confirm that the conversion of structurally homogeneous stands into uneven-aged structures cannot be reduced to a single thinning or marking operation. The optimised harvest schedule did not simply remove trees from all diameter classes proportionally. Instead, the model concentrated harvests in structurally surplus medium and

https://doi.org/10.17221/44/2026-JFS

upper classes while completely avoiding harvests in deficit lower classes. This confirms that uneven-aged conversion is a distributional control problem in which the state of the stand is represented by the whole diameter-class vector rather than by a single aggregate variable such as age, standing volume, or basal area.

This finding is consistent with the classical interpretation of uneven-aged optimisation by Haight (1985), who showed that dynamic economic models may produce different conclusions from static models because current harvest decisions alter future stand structure and future harvest opportunities. Our results support this view directly. The model selected relatively different harvest vectors across cycles, indicating that an optimal conversion pathway is path-dependent. The first cycle reduced surplus in the middle classes, while later cycles shifted harvest pressure toward upper classes as the transition matrix redistributed residual trees across diameter classes.

The results also support the matrix-model tradition initiated by Buongiorno and Michie (1980). Their work demonstrated that uneven-aged stands can be effectively represented through diameter-class transition systems. Our model confirms the practical relevance of this approach for conversion planning: the transition matrix was sufficient to generate a dynamic link between post-harvest residual structure and future stand states. However, our results also show that although harvesting decisions regulate stand structure, attainment of a target uneven-aged distribution ultimately depends on the biological processes represented within the transition matrix. In particular, recruitment and mortality determine whether the desired structure can be sustained over time, a principle that is well established in uneven-aged forest management literature. This aligns with recent findings emphasising that recruitment uncertainty is the dominant constraint in uneven-aged matrix modelling (O'Hara 2023).

The hypothetical example confirms the usefulness of the Liocourt curve as a transparent regulatory benchmark. The target distribution enabled the model to identify structural surpluses and deficits in individual diameter classes and to translate these deviations into harvest decisions. In this sense, the model supports the practical value of Liocourt-type regulation and the BDq method as a decision-support framework. Recent empirical work in European mixed forests similarly shows that BDq performance depends strongly on regeneration and growth conditions (Brzeziecki, Neroj 2024).

At the same time, the results clearly reject a simplistic interpretation of the Liocourt distribution as an automatically attainable biological equilibrium. The lower classes remained substantially below their target values even after three optimised management cycles. In the numerical example, the first diameter class reached only 97.012 trees per ha compared with the target value of 182.824 trees per ha. This persistent deficit occurred despite the fact that no harvest was allowed or selected in the lower classes.

The model therefore demonstrates that convergence toward a Liocourt-type target structure is strongly constrained by recruitment dynamics. This finding should not be interpreted as a contradiction of contemporary uneven-aged silvicultural theory. Modern selection-system research recognises that sustainable stand structures emerge from the combined effects of recruitment, growth, mortality, and harvesting, rather than from harvesting interventions alone (Kerr 2014; O'Hara 2023; Picard, Gasparotto 2016). Harvesting can regulate the distribution of existing trees among diameter classes, but it cannot generate the continuous inflow of individuals required to maintain lower diameter classes. Consequently, successful conversion toward uneven-aged structures depends not only on harvest regulation but also on sufficient regeneration and subsequent recruitment into the managed diameter distribution.

Our results therefore reinforce an established principle of uneven-aged forest management: target diameter distributions should be evaluated together with the biological processes that sustain them. Within the present optimisation framework, recruitment is not merely a source of uncertainty but a fundamental determinant of whether a target structure is feasible under given ecological conditions. This result strongly supports the arguments of Kerr (2014), who emphasised that de Liocourt's original work has often been overinterpreted and that the q-ratio should not be treated as a universal law of selection forests. It also confirms the interpretation by Picard and Gasparotto (2016), who showed that the modern formalisation of Liocourt's law has a more complex historical and mathematical background than is often assumed. Our results

https://doi.org/10.17221/44/2026-JFS

are consistent with their conclusion: the Liocourt curve is a useful theoretical structure, but it is not a guaranteed outcome of stand dynamics.

The study therefore refines the use of Liocourt's law in planning. It shows that the Liocourt curve should be used as a regulatory hypothesis tested against biological dynamics, not as a fixed biological truth imposed on the forest. This distinction is important for operational forestry. If the target *q*-ratio and basal area are set without verifying recruitment feasibility, the optimisation may produce harvest prescriptions that appear structurally rational but cannot deliver the desired future stand structure.

One of the strongest results of the study is the dominant influence of recruitment on successful conversion. In the numerical example, harvest was excluded from deficit lower classes, yet the first diameter class remained far below the target. This confirms that the limiting factor was not excessive harvest pressure on small trees but insufficient inflow into the lower diameter classes.

This matches recent findings that regeneration inflow is the key determinant of structural stability in uneven-aged systems (Brzeziecki, Neroj 2024).

The scenario analysis reinforces this interpretation. The best-performing scenarios were associated with high recruitment, while scenarios with low recruitment generated large terminal deviations from the target structure. The best scenario achieved a terminal *RMSE* of 3.861, whereas poorer combinations of *q*-ratio, basal area, growth, mortality, and recruitment produced much higher deviations.

This confirms a widely reported fact in uneven-aged modelling: recruitment and ingrowth are often the most uncertain and most influential components of long-term projections. Hao et al. (2005) showed that transition matrix models for uneven-aged mixed forests must explicitly account for ingrowth and upgrowth processes.

The results also support the broader modelling conclusions of Virgilietti and Buongiorno (1997), who treated matrix models as flexible tools for representing forest dynamics from management data. Our analysis suggests that such models are especially useful when recruitment can be empirically calibrated from repeated inventories. Without such calibration, the lower part of the diameter distribution becomes the weakest component of the optimisation framework.

The scenario results indicate that the *q*-ratio has a strong effect on structural feasibility. Higher *q*-ratios generate steeper target distributions and therefore require substantially more trees in lower diameter classes. If recruitment is limited, such targets become difficult or impossible to achieve. In the scenario batch, robust performance was associated with lower target basal area and moderate to lower *q*-ratio values. The combination $q = 1.15$ and $B = 14\ m^2 \cdot ha^{-1}$ performed best according to average terminal *RMSE*, while higher *q*-ratios increased structural sensitivity. The superior performance of lower target basal areas in the optimisation results should be interpreted with caution. Within the present model, lower basal area reduced structural constraints and facilitated convergence toward the target diameter distribution, particularly by reducing competition and increasing the potential for recruitment into lower diameter classes. However, lower basal area generally implies lower standing volume and may reduce total volume production if maintained over extended periods. Consequently, the result should not be interpreted as a recommendation to maintain a basal area of $14\ m^2 \cdot ha^{-1}$ as a permanent management target. From a silvicultural perspective, such conditions may represent a transitional phase during conversion from a structurally homogeneous stand toward a regulated uneven-aged structure. Once sufficient regeneration and recruitment have been established, higher basal areas may become compatible with maintaining structural stability and sustainable yield. Therefore, the optimal target basal area depends on species composition, site productivity, regeneration dynamics, and management objectives rather than being a universal constant (O'Hara 2023; Kerr 2014).

This result supports Chevrou's (1990) argument that the *q*-ratio is not a fixed biological constant but depends on management conditions, thinning regime, and structural assumptions. Our results extend this argument into a conversion-planning context. The *q*-ratio should be treated as a decision parameter subject to scenario testing, not as an externally fixed rule.

The findings also partially qualify the traditional BDq approach. The BDq method is operationally attractive because it simplifies structural regulation into basal area, maximum diameter, and *q*-ratio. However, our results show that these parameters interact strongly with recruitment and growth.

https://doi.org/10.17221/44/2026-JFS

A BDq target can be mathematically defined even when it is biologically difficult to achieve. Therefore, BDq should be coupled with dynamic feasibility analysis before being used as a marking guide for conversion stands.

The quadratic objective function explicitly combines economic returns from harvesting with penalties for structural deviation. The results demonstrate that these two objectives may conflict. Short-term economic optimisation favours harvesting trees with positive net value, particularly in larger or commercially attractive diameter classes. Structural optimisation, by contrast, may require retaining certain trees or limiting harvest intensity to maintain future distributional balance.

This result supports Haight's (1985) conclusion that static and dynamic optimisation can produce different management prescriptions. It also aligns with López-Torres et al. (2016), who optimised uneven-aged *Pinus nigra* management between stable structures and showed that economic results depend on the selected structural pathway. Our model confirms this principle in the specific context of conversion from homogeneous to uneven-aged structure: the economically attractive removal of surplus trees must be constrained by the long-term objective of structural convergence.

The structural penalty parameter controls this trade-off. A low penalty favours harvest revenues but allows larger deviations from the target Liocourt distribution. A high penalty improves structural convergence but reduces immediate economic return. This confirms that the conversion problem is inherently multi-objective, even when formulated as a single aggregated objective function.

The results also suggest that the economically optimal steady-state structure may not coincide with a classical Liocourt curve. This is consistent with Pukkala et al. (2010), who found that optimised uneven-sized stand structures depend on economic assumptions and may differ from traditional silvicultural expectations. Our results therefore support the argument that Liocourt or BDq targets should be evaluated against economic criteria rather than assumed to be economically optimal.

The scenario analysis indicates that a lower target basal area improved structural convergence. Scenarios with B = 14 $m^2 \cdot ha^{-1}$ performed better than scenarios with higher target basal area. This result is biologically plausible because lower target basal area reduces competition and may improve the light conditions necessary for regeneration and ingrowth. It also reduces the number of trees required to satisfy the target distribution.

This result is consistent with the broader close-to-nature silviculture literature, which emphasises that stocking level affects regeneration, stability, and adaptive capacity. Brang et al. (2014) argued that close-to-nature silviculture may support adaptation by increasing structural diversity and reducing vulnerability, but also noted that adaptation requires active management rather than passive maintenance of high stocking. Our results support this interpretation: maintaining or imposing a high target basal area may make structural conversion more difficult when lower diameter classes are already deficient.

However, this finding should not be interpreted as a universal recommendation for low basal area. It reflects the particular initial structure and transition assumptions of the hypothetical example. In more productive sites or under stronger recruitment, higher basal area targets may be feasible. The result therefore confirms the need for local calibration of BDq parameters.

The study confirms the usefulness of a matrix-based quadratic programming framework for transparent decision support. The model produces interpretable outputs: harvest by diameter class and cycle, basal area before and after intervention, terminal deviation from target structure, and scenario-level performance indicators. This makes the framework suitable for communication between forest planners, researchers, and practitioners.

This supports the long-standing use of transition matrix models in uneven-aged forestry, as developed by Buongiorno and Michie (1980) and later extended in mixed-species contexts by Hao et al. (2005). The present study adds that the same framework can be directly connected with modern quadratic programming solvers, making it operationally useful for testing alternative conversion strategies.

At the same time, the results reveal that model transparency does not eliminate biological uncertainty. The model is only as reliable as the transition matrix, recruitment assumptions, mortality estimates, and economic parameters. This is especially important because the scenario analysis showed strong sensitivity to recruitment and mortality. Therefore, repeated inventories, permanent sample plots, remote sensing-assisted monitoring, and local

https://doi.org/10.17221/44/2026-JFS

calibration are necessary before such a model can be used for real operational prescriptions.

The results provide a nuanced view of CCF as an adaptation strategy. They support the idea that structurally diverse forests can be a desirable management objective, but they also show that structural diversity is not achieved automatically. Conversion requires a biologically feasible pathway, sufficient recruitment, and management constraints that prevent short-term economic extraction from undermining long-term structure.

This partially supports the conclusions of Brang et al. (2014), who identified close-to-nature silviculture as potentially suitable for adaptation of temperate European forests to climate change. Our results confirm the relevance of structural diversification but add a planning caveat: adaptation-oriented structures must be dynamically attainable under site-specific recruitment and mortality conditions.

The model also suggests that CCF conversion should be evaluated through robustness rather than through a single deterministic forecast. Climate change may alter mortality, growth transitions, and regeneration success. Therefore, future applications should extend the present deterministic model toward stochastic or robust optimisation.

The study is based on a hypothetical stand and simplified transition dynamics. The results, therefore, demonstrate model behaviour and theoretical implications rather than provide site-specific management recommendations. The transition matrix used in the numerical example is deterministic and does not include species composition, climate-dependent mortality, browsing pressure, seed limitation, or spatial heterogeneity.

The model also uses a simplified economic representation. Real harvest revenues depend on species, quality, assortment structure, logging technology, accessibility, terrain, and market conditions. Similarly, the structural penalty parameter is normative and requires calibration according to management priorities.

Despite these limitations, the results are useful because they isolate the core mechanism of uneven-aged conversion. They show that harvest optimisation can reduce structural surplus but cannot create missing lower diameter classes. This conclusion is robust and directly relevant for practical CCF conversion planning.

Future research should focus on four main directions. First, transition matrices should be calibrated using repeated inventory data, ideally including species-specific growth, mortality, and ingrowth. Second, recruitment functions should be linked to basal area, canopy openness, browsing pressure, and site conditions rather than treated as fixed constants. Third, the model should be extended to stochastic mortality and disturbance risk. Fourth, carbon, biodiversity, and habitat objectives should be integrated into the optimisation framework. Such extensions would allow the model to move from a theoretical demonstration toward a full decision support system for climate-adaptive Continuous Cover Forestry.

Although the present study uses a hypothetical stand, the proposed framework can be calibrated using repeated forest inventory measurements. Diameter-class frequencies obtained from permanent sample plots may be used to estimate transition probabilities, mortality rates, and recruitment intensity for each management cycle. Growth and mortality parameters can be incorporated into the transition matrix, while recruitment rates may be estimated from observed ingrowth into the smallest inventoried diameter class. Economic parameters, including timber prices, harvesting costs, and discount rates, can be adapted to local management conditions. Such calibration would allow the framework to support practical planning of conversion toward uneven-aged structures under site-specific ecological and economic conditions.

CONCLUSION

The presented study developed and analysed a multi-period optimisation framework for the conversion of structurally homogeneous forest stands toward uneven-aged forest structures regulated through Liocourt-type diameter distributions. The proposed model combines forest growth dynamics, structural regulation, and economic optimisation within a unified quadratic programming framework and demonstrates that uneven-aged forest conversion cannot be understood as a purely silvicultural or harvest scheduling problem, but rather as a long-term dynamic process of structural regulation.

The results clearly indicate that successful conversion toward uneven-aged stand structures is driven primarily by recruitment processes and by the dynamics of lower diameter classes. While harvesting can effectively reduce structural surplus

https://doi.org/10.17221/44/2026-JFS

in medium and upper diameter classes, it cannot compensate for deficits in small-diameter trees. Consequently, the long-term success of conversion strategies depends fundamentally on the stability of regeneration, recruitment intensity, and the transition of trees between lower diameter classes. This finding confirms that regeneration processes represent the critical biological mechanism maintaining uneven-aged stand continuity.

The study further demonstrates that Liocourt distributions should not be interpreted as universal biological equilibrium states. Instead, they should be viewed as regulatory optimisation targets that can guide structural development under specific ecological and economic conditions. Different combinations of growth, mortality, recruitment, and harvest intensity may lead to structurally different developmental trajectories even under identical q-ratios. The classical inverted-J diameter distribution therefore represents an idealised management objective rather than an inevitable or universally stable outcome of uneven-aged forest dynamics.

From a methodological perspective, the study confirms that multi-period quadratic programming provides a robust and mathematically consistent framework for long-term structural forest optimisation. The quadratic penalty formulation used in the objective function creates a stable optimisation environment with unique local optima and enables explicit balancing between economic objectives and structural regulation. In contrast to sequential or short-term optimisation approaches, the multi-period framework allows optimisation of the entire developmental trajectory of the stand and therefore captures the delayed structural consequences of harvesting decisions.

At the same time, the analyses reveal that recruitment and mortality represent the dominant uncertainty sources in uneven-aged forest management models. Small changes in these parameters substantially influence long-term structural stability, convergence toward target distributions, and achievable harvest intensities. This sensitivity highlights the necessity of continuous model calibration using repeated forest inventory data and indicates that robust optimisation approaches may become increasingly important under conditions of climate change and increasing disturbance frequency.

An important outcome of the study is the explicit identification of trade-offs between economic return and long-term structural stability. Higher harvest intensities and economically optimised harvesting strategies often increase structural deviations from target distributions, whereas stronger structural regulation reduces short-term economic efficiency in favour of long-term stand stability. The proposed framework therefore enables transparent quantification of these trade-offs and creates opportunities for multi-objective forest planning integrating production, ecological stability, and resilience objectives.

Despite the promising results, the study also identifies several important directions for future research. Future model extensions should integrate disturbance dynamics, stochastic mortality processes, carbon sequestration functions, and spatial optimisation. Incorporation of climate-sensitive growth and recruitment functions, as well as explicit modelling of disturbance risk, will likely become essential for realistic long-term planning under changing environmental conditions. Similarly, integration with modern remote sensing systems and continuous forest inventory approaches could significantly improve parameter calibration and operational applicability.

Overall, the proposed optimisation framework provides a theoretically consistent basis for the future development of climate-adaptive decision support systems for Continuous Cover Forestry and uneven-aged forest management. By combining structural regulation, stand dynamics, and economic optimisation within a single analytical framework, the model offers substantial potential for supporting adaptive and resilient forest management strategies in conditions of increasing ecological uncertainty.

REFERENCES

Brang P., Spathelf P., Larsen J.B., Bauhus J., Boncina A., Chauvin C., Drossler L., Garcia-Guemes C., Heiri C., Kerr G., Lexer M.J., Mason B., Mohren F., Muhlethaler U., Nocentini S., Svoboda M. (2014): Suitability of close-to-nature silviculture for adapting temperate European forests to climate change. Forestry, 87: 492–503.

Brzeziecki B., Neroj B. (2024): The BDq method as a tool supporting the development of diversified structure in silver fir *Abies alba* Mill. stands in the Bieszczady Mountains, Southeastern Poland. Sylwan, 168: 12–22.

Buongiorno J., Michie B.R. (1980): A matrix model of uneven-aged forest management. Forest Science, 26: 609–625.

https://doi.org/10.17221/44/2026-JFS

Chevrou R.B. (1990): De Liocourt's law and the truncated law. Canadian Journal of Forest Research, 20: 1933–1946.

Constantino M., Martins I., Borges J.G. (2008): A new mixed-integer programming model for harvest scheduling subject to maximum area restrictions. Operations Research, 56: 542–551.

Haight R.G. (1985): A comparison of dynamic and static economic models of uneven-aged stand management. Forest Science, 31: 957–974.

Haight R.G., Brodie J.D., Adams D.M. (1985): Optimizing the sequence of diameter distributions and selection harvests for uneven-aged stand management. Forest Science, 31: 451–462.

Hao Q., Meng F.R., Zhou Y., Wang J.R. (2005): A transition matrix growth model for uneven-aged mixed-species forests in the Changbai Mountains, northeastern China. New Forests, 29: 221–231.

Hyytiäinen K., Haight R.G. (2012): Optimizing continuous cover forest management. In: Pukkala T., von Gadow K. (eds): Continuous Cover Forestry. Dordrecht, Springer: 195–227.

Kerr G. (2014): The management of silver fir forests: De Liocourt (1898) revisited. Forestry, 87: 29–38.

López-Torres I., Ortuño Pérez S., García Robredo F., Fullana Belda C. (2016): Optimizing the management of uneven-aged *Pinus nigra* stands between two stable positions. iForest – Biogeosciences and Forestry, 9: 599–607.

O'Hara K.L. (2023): Virtual issue on uneven-aged forestry. Forestry: An International Journal of Forest Research.

Picard N., Gasparotto D. (2016): Liocourt's law for tree diameter distribution in forest stands. Annals of Forest Science, 73: 751–755.

Pukkala T., Lähde E., Laiho O. (2010): Optimizing the structure and management of uneven-sized stands of Finland. Forestry, 83: 129–142.

Schwartz J.W., Nagel L.M., Webster C.R. (2005): Effects of uneven-aged management on diameter distribution and species composition of northern hardwoods in Upper Michigan. Forest Ecology and Management, 211: 356–370.

Virgilietti P., Buongiorno J. (1997): Modeling forest growth with management data: A matrix approach for the Italian Alps. Silva Fennica, 31: 27–42.

Received: May 25, 2026
Accepted: July 21, 2026
Published online: July 30, 2026