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Additive volume-equation systems for *Pinus ayacahuite* and *Pinus douglasiana* in temperate forests of the Sierra Norte, Oaxaca, Mexico

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Abstract: Volume models are essential tools for quantifying timber stocks and optimising forest utilisation. This study aimed to develop additive volume systems based on one- and two-entry simultaneous equations for *Pinus ayacahuite* Ehrenb. ex Schltdl. and *Pinus douglasiana* Martínez. Destructive sampling of 55 *P. ayacahuite* trees and 65 *P. douglasiana* trees was conducted in the communal forest of Ixtlán de Juárez, Oaxaca, southern Mexico. The additive systems were fitted using non-linear seemingly unrelated regression to estimate tree-volume components: stem and branch volumes, with whole-tree volume being the sum of both. The systems were evaluated using the relative ranking method, considering statistical indicators of accuracy, variability, and relative errors. Additionally, the predictive capacity of the equations was assessed through linear regression between observed and predicted values for each volume component, and the biological consistency was verified. The results indicate that two-entry additive systems provide greater accuracy in estimating stem, branch, and whole-tree volumes for both species. These equations are based on the Schumacher-Hall model, and their recommended range of application for both species is for diameter at breast height (*DBH*) between 9 cm and 75 cm, and for total height (*H*) between 9 m and 34 m. Therefore, their application is recommended for forest inventories and the planning of sustainable forest management.

Keywords: forest inventory; forestry; regression; simultaneous fitting; volume tables; whole-tree volume

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Accurate estimation of timber stocks in production forests is essential for sustainable management, as it enables the optimisation of silvicultural prescriptions, the projection of expected forest products, and the study and understanding of ecosystem dynamics (Avery, Burkhart 2002; Pretzsch 2009; Vargas-Larreta et al. 2017). This estimation generally uses indirect methods through allometric models that employ dendrometric variables such as diameter at breast height (*DBH*) and total height (*H*) (Clutter et al. 1983; Burkhart, Tomé 2012). These models offer a practical means of reliably estimating standing tree volume with minimal field effort (Avery, Burkhart 2002).

Allometric volume models are essential tools in forest planning, allowing for the quantification of timber stocks, assessment of management practices, estimation of biomass and above-ground carbon, and support for environmental monitoring and research programmes (Pretzsch 2009; Burkhart, Tomé 2012). They are also used to assess pest-affected areas, monitor harvesting operations, verify cutting intensities, and project volume growth. Thus, they form the technical cornerstone of silviculture and forest management (Ramírez-Martínez et al. 2016). In this context, accurate estimations of timber volume are crucial for sustainably managing forest resources and monitoring the impacts of climate change on forest ecosystems.

To improve estimation accuracy, models are recommended to be calibrated at the species level (Vallet et al. 2006; Tabacchi et al. 2011; Bornand et al. 2023; Santiago-García 2025). Local-scale volume models for natural forests are preferred over generic models, as volume estimates should be calculated according to forest type, respecting species composition and structure (de Souza et al. 2024; Santiago-García 2025). Moreover, when estimating the volume of different tree components (stem and branches), it is essential to consider the property of additivity, which ensures mathematical consistency between the parts and whole-tree volume by recognising the inherent correlations among the measured components (Parresol 2001; Dong et al. 2014).

Additive volume models enable detailed disaggregation of volume by structural component and are especially useful in contexts requiring consistency between partial and total estimates. In the forestry literature, non-linear seemingly unrelated regression (NLSUR) has proven to be an effective tool for modelling both biomass (Parresol 2001; Brandesis et al. 2006; Dong et al. 2014; Zhao et al. 2017) and volume in an additive and compatible manner (Vargas-Larreta et al. 2017; Behling et al. 2019; Wang et al. 2024). This methodology is flexible and powerful, as it allows each component model to use its own set of independent variables and weighting functions to address heteroscedasticity, which reduces estimator variance and improves the overall accuracy of the system (Parresol 2001; Dong et al. 2014).

Pinus ayacahuite Ehrenb. ex Schltdl. and *Pinus douglasiana* Martínez are representative species of the temperate forest of Ixtlán de Juárez in the Sierra Norte of Oaxaca, Mexico, and possess high commercial value (Santiago-García et al. 2020b). However, most local volume equations used in forest management plans estimate only stem volume (Ramírez-Martínez et al. 2016; Santiago-García et al. 2020a). Current forestry regulations (NOM-152-SEMARNAT-2023) stipulate that volumetrics must be estimated at the whole-tree level, including branches. Therefore, models that comprehensively account for the volume of all tree components are necessary.

The use of outdated volume equations may result in biased estimates, potentially causing economic losses for forest producers or greater environmental impacts by necessitating the felling of more trees to meet authorised harvest volumes (Melchor et al. 2010). Ramírez-Martínez et al. (2016) emphasise that volume models must be developed specifically for each species and location to avoid errors in stock estimation and the application of silvicultural treatments.

In the Sierra Norte of Oaxaca, Mexico, volume equations have been developed for various species with a regional focus (Vargas-Larreta et al. 2017); however, the community of Ixtlán de Juárez – recognised as a model of community-based forestry (Santiago-García et al. 2022) – requires tools adapted to local conditions. These tools must reliably estimate timber volume based on the specific characteristics of the landholding to fulfil production, conservation, and monitoring objectives.

In the absence of local tools for compatibly quantifying whole-tree volume in the communal forests of Ixtlán de Juárez, Oaxaca, this study aimed to develop additive volume systems for *P. ayacahuite* and *P. douglasiana*, to improve the precision of timber volume estimation and provide robust, up-to-date tools to support the planning and execution of sustainable forest management.

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MATERIAL AND METHODS

Study area. The study was conducted in the community of Ixtlán de Juárez, located northwest of the city of Oaxaca, in the Sierra Norte de Oaxaca, Mexico. The area is located between the following coordinates: 17°16'48''N–17°40'48''N, 96°15'00''W–96°31'48''W. (Figure 1). The communal forest covers an area of 19 310 ha, and its predominant vegetation corresponds to pine-oak forest (*Pinus-Quercus*), typical of the temperate mountainous zones of the region (Santiago-García et al. 2022).

According to the Köppen climate classification, modified by García (1987), the climate of the region is classified as a humid temperate climate with summer rainfall, characteristic of mid- to high-altitude areas in the Sierra Norte.

Sampling. The sampling involved selecting 55 individual *P. ayacahuite* trees and 65 *P. douglasiana* trees, covering a wide range of age, diameter, and height classes. These trees were considered ideal for fitting allometric models, as they were healthy and vigorous, with straight stems and no forks, located in harvested areas within the communal forest.

The sampling strategy followed the recommendations of Torres and Magaña (2001), who propose a minimum of 50 trees per species to ensure reliable allometric equation fitting. This methodology is also consistent with recent studies, such as those by Lee et al. (2017) and Santiago-García (2025), which employ similar sample sizes for developing volume models.

To ensure the representativeness of the data, the diameter distribution of the sample was verified to follow a normal distribution (Figure 2), using the Shapiro-Wilk test (Shapiro, Wilk 1965). This supported the statistical validity of the data used for the model fitting.

Data collection. The selected trees were felled using directional felling techniques to avoid damaging the remaining vegetation. Once on the ground, each tree was segmented into stem, top, and branches. The stem logs were measured according to the commercial lengths of 2.62 m and 1.25 m, complemented with shorter sections at the tree base to better capture stem form. Additionally, tops and branches with a basal diameter of 5 cm or more were included (Figure 3), following the protocol described by Vargas-Larreta et al. (2017). All diameters were measured over bark, ensuring consistency with methods commonly used in volume studies.

Table 1 summarises the descriptive statistics of the sample used to develop the additive volume systems for *P. ayacahuite* (n = 55) and *P. douglasiana* (n = 65). Although both species presented similar average values in their mensurational variables, *P. douglasiana* exhibited greater variability in the volume of structural components.

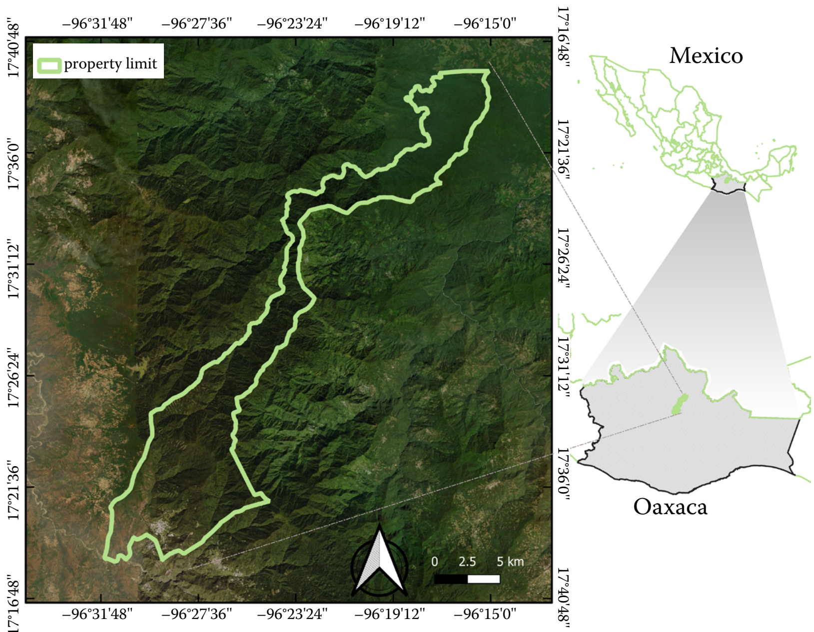

Figure 1. Location of the communal forest of Ixtlán de Juárez, Oaxaca, Mexico

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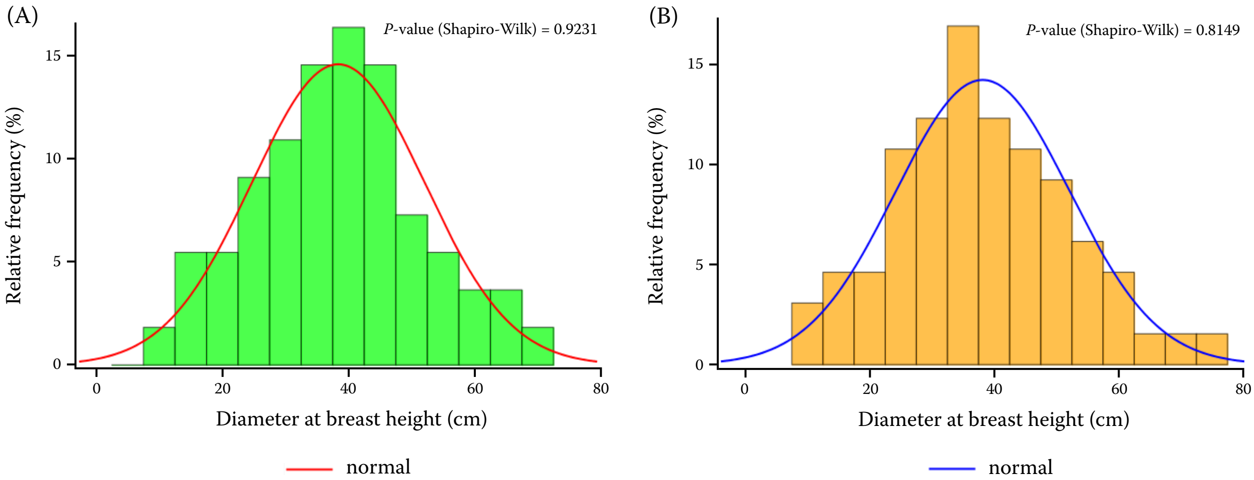

Figure 2. Diameter distribution of the sample used for fitting the additive volume systems: (A) *Pinus ayacahuite*, (B) *Pinus douglasiana*

Volume calculation. Volume was calculated by sections, using Smalian's formula for intermediate or neiloid-shaped logs, see Equation (1); the cone formula for the top section was calculated according to Equation (2) (Özçelik et al. 2016; Vargas-Larreta et al. 2017):

$$V_s = \frac{\pi}{40\ 000}\left(\frac{S_0 + S_1}{2}\right)l \quad (1)$$

$$V_s = \frac{\pi}{40\,000}\left(\frac{S_0}{3}\right)l \quad (2)$$

where:
V_s – volume of the section (m^3);
S_0 – initial cross-sectional area (m^2);
S_1 – final cross-sectional area (m^2);
l – length of the log (m).

Total stem volume over bark was obtained by summing the volumes of all sections (excluding the stump) and the top. Branch volume was calculated using the same procedure, considering sections with a basal diameter equal to or greater than 5 cm. Finally, whole-tree volume (m^3) was obtained by summing the stem volume over bark and the branch volume (Corral-Rivas et al. 2017; Vargas-Larreta et al. 2017).

Additive volume equation systems. The development of additive equation systems, both for one- and two-entry models, began by selecting individual volume equations that demonstrated the best statistical performance and biological plausibility. This selection was based on a preliminary fitting of multiple models commonly used in forest biometrics (Avery, Burkhart 2002; Diéguez-Aranda et al. 2003). The best-fitting model for one-entry systems was the Berkhout model, as shown in Equation (3), while for two-entry systems, the Schumacher-Hall model, as shown in Equation (4), was selected due to its statistical robustness and compatibility with tree growth patterns:

$$V = \beta_0 \times DBH^{\beta_1} \quad (3)$$

$$V = \beta_0 \times DBH^{\beta_1} \times H^{\beta_2} \quad (4)$$

where:
V – volume (m^3);
β_n – model parameters;
DBH – diameter at breast height (cm);
H – total tree height (m).

In the additive volume systems for both one-entry Equations (5–7) and two-entry models according to Equations (8–10), simultaneous fitting of stem, branch, and whole-tree volumes was conducted (Parresol et al. 2001):

$$V_{st} = \beta_0 \times DBH^{\beta_1} \quad (5)$$

$$V_{br} = e^{-\alpha_0} \times DBH^{\alpha_1} \quad (6)$$

$$V_{wt} = \beta_0 \times DBH^{\beta_1} + e^{-\alpha_0} \times DBH^{\alpha_1} \quad (7)$$

$$V_{st} = \beta_0 \times DBH^{\beta_1} \times H^{\beta_2} \quad (8)$$

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$$V_{br} = e^{-\alpha_0} \times DBH^{\alpha_1} \quad (9)$$

$$V_{wt} = \beta_0 \times DBH^{\beta_1} \times H^{\beta_2} + e^{-\alpha_0} \times DBH^{\alpha_1} \quad (10)$$

where:

V_{st} – stem volume over bark (m^3);

V_{br} – branch volume over bark (m^3);

V_{wt} – whole-tree volume over bark (m^3).

Data analysis. The statistical fitting of the additive volume systems was carried out using NLSUR, with the MODEL procedure of the SAS/ETS software package (Version 9.4, 2013; SAS 2013). This methodology, based on the approach proposed by Zellner (1962), allows the simultaneous fitting of multiple related equations, assuming that the errors are independent over time while permitting contemporaneous correlations across equations.

NLSUR use is appropriate for developing additive volume systems because it ensures compatibility between the individual components (stem and branch volumes) and the whole-tree volume, respecting the biological constraints inherent to the tree structure (Parresol 2001). Moreover, this technique enhances the efficiency of the estimators by exploiting the information contained in the error covariances across equations, resulting in more precise parameter estimates compared to separate estimations (Zellner 1962; Dong et al. 2015). During model fitting, explicit additive constraints were specified in the MODEL procedure to ensure that the total volume equalled the sum of its components.

For the goodness-of-fit analysis, accuracy, variability, and relative error statistics were considered (Table 2), including the following: adjusted coefficient of determination (R^2_{adj}), root mean square error (*RMSE*), mean absolute error (*MAE*), Akaike information criterion (*AIC*), total relative error (*TRE*), and mean percent standard error (*MPSE*) (Poudel, Cao 2013; Özçelik et al. 2016; Zeng et al. 2017; Ogana, Ercanli 2022).

Heteroscedasticity correction was conducted by applying a variance-weighting function to the residuals, following the specification of Balboa-Murias et al. (2006), Quiñonez-Barraza et al. (2014), and Dong et al. (2015), as shown in Equation (11).

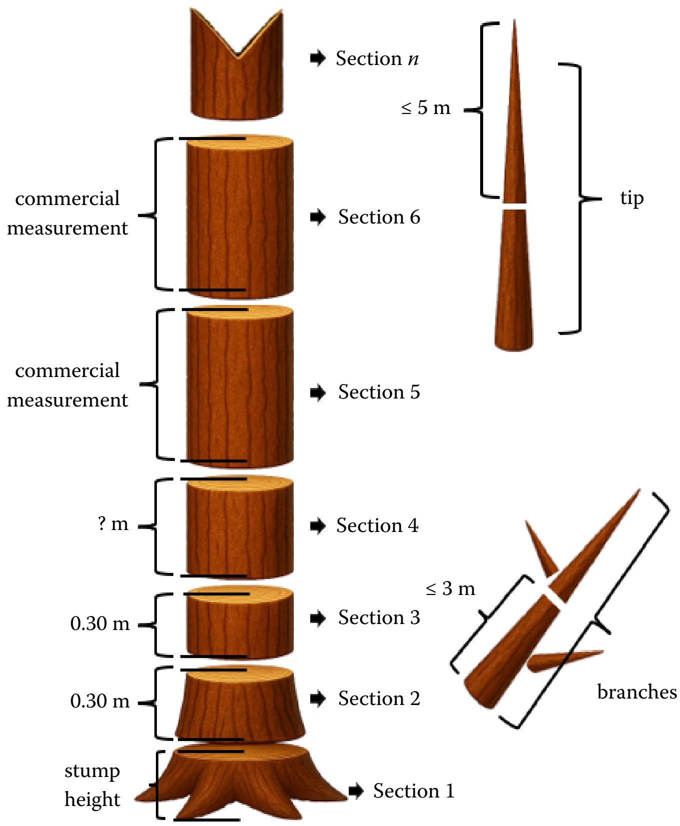

Figure 3. Schematic representation of data collection and segmentation of sampled trees into structural components

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Table 1. Descriptive statistics of the sample used for constructing the additive volume systems

Variable	*n*	Mean	Minimum	Maximum	Variance	SD
Pinus ayacahuite						
H		26.72	9.88	33.37	22.91	4.79
DBH		38.36	9.40	71.00	187.28	13.69
V_{st}	55	1.60	0.04	4.81	1.21	1.10
V_{br}		0.09	0.00	0.45	0.01	0.10
V_{wt}		1.69	0.04	5.26	1.42	1.19
Pinus douglasiana						
H		24.90	9.74	33.09	31.83	5.64
DBH		38.16	10.35	74.00	196.52	14.02
V_{st}	65	1.71	0.05	5.92	1.81	1.34
V_{br}		0.17	0.00	1.35	0.06	0.24
V_{wt}		1.87	0.05	6.80	2.43	1.56

H – total tree height (m); *DBH* – diameter at breast height (cm); V_{st} – stem volume over bark (m^3); V_{br} – branch volume over bark (m^3); V_{wt} – whole-tree volume over bark (m^3); *n* – sample size; SD – standard deviation

$$resid.V_c = \frac{resid.V_c}{\left[\left(x_i\right)^{\omega}\right]^{0.5}} \quad (11)$$

where:

V_c – volume of the component;

x_i – independent variables;

ω – weighting function parameter.

The ω parameter was estimated for each volume equation, either iteratively or through logarithmic linear regression of the residuals from the unweighted model: $ln\ (\hat{e}_i^2) = \alpha + \omega \times ln(x_i)$ (Harvey 1976; Balboa-Murias et al. 2006). The variable x_i corresponded to *DBH* in the one-entry system and to *DBH* × *H* in the two-entry system, where V_c represents the volume of the component: stem, branches, or whole tree.

The homoscedasticity assumption was verified using White's test (White 1980). To evaluate the predictive performance of the equations, linear regression was conducted between observed (*y*-axis) and predicted (*x*-axis) volume values, using the 1 : 1 line as a reference (Piñeiro et al. 2008). Additionally, the models were graphically assessed by superimposing the predicted values on observed data and by verifying the biological consistency between the whole-tree volume curve and the corresponding stem-volume curves.

Model selection. The best-fitted models, both single- and two-entry, were selected using the relative

Table 2. Statistical indicators for evaluating the goodness-of-fit of the additive volume systems

Statistic	Expression
Adjusted coefficient of determination	$R^2_{adj} = 1 - \frac{(n-1)}{(n-p)} \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2}$
Root mean square error	$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-p}}$
Mean absolute error	$MAE = \frac{1}{n}\sum_{i=1}^{n}\lvert y_i - \hat{y}_i \rvert$
Akaike information criterion	$AIC = 2p + n\,ln\left[\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}\right]$
Total relative error	$TRE = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)}{\sum_{i=1}^{n}\hat{y}_i} \times 100$
Mean percent standard error	$MPSE = \frac{\sum_{i=1}^{n}\left\lvert\frac{(y_i - \hat{y}_i)}{\hat{y}_i}\right\rvert}{n} \times 100$

$\hat{y}_i$, y_i, $\bar{y}_i$ – predicted, observed, and mean values of volume, respectively; *n* – total number of observations used for model fitting; *p* – number of model parameters

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ranking method proposed by Poudel and Cao (2013) and applied in recent studies by Ogana and Ercanli (2022), He et al. (2022), and Jha et al. (2023). This approach enables an objective comparison of models based on multiple goodness-of-fit statistics. The relative rank for each model i was calculated using Equation (12):

$$R_i = 1 + (m-1) \times \frac{(S_i - S_{\min})}{S_{\max} - S_{\min}} \tag{12}$$

where:

R_i – relative rank of model i (i = 1, 2, ..., m);

m – number of models evaluated;

S_i – value of the goodness-of-fit statistic for model i;

$S_{\min}$, $S_{\max}$ – the minimum and maximum values of the statistic S_i, respectively.

This procedure was applied to each of the statistics listed in Table 2. The relative ranks obtained by each model for the different fit criteria were then summed. The model with the lowest total sum of relative ranks was considered the best fitted, as it reflected the most consistent performance across all evaluated indicators.

RESULTS

Additive systems for *P. ayacahuite*. Table 3 presents the goodness-of-fit statistics and estimated parameters of the single- and two-entry additive volume systems developed for *P. ayacahuite*. Overall, the evaluated equations demonstrated good predictive performance, with adjusted coefficients of determination (R^2_{adj}) ranging from 0.797 to 0.993, indicating a high capacity to explain the variability of the data. Among the modelled components, the equations corresponding to branch volume showed the lowest R^2_{adj} values, explaining approximately 80% of the total observed variability in both single- and two-entry systems. All estimated parameters were statistically significant (P < 0.0001) at a significance level of α = 0.05, supporting the validity of the model fits.

According to the relative rank-sum criterion, the two-entry additive system achieved the lowest total rank sum and was thus considered the most appropriate approach for estimating component-wise tree volume. This system enables the prediction of stem volume, branch volume, and whole-tree volume (V_{wt}) based on diameter at breast height (*DBH*) and total height (*H*) (Table 3).

The predicted values adequately followed the trend of the observed data, confirming the high predictive capability of the models. Additionally, the comparison between stem volume and whole-tree volume exhibited a biologically consistent pattern, with stem volume systematically lower than V_{wt}. This pattern ensures accurate and consistent estimates in scenarios in which only *DBH* is available and when *DBH* and *H* are used jointly (Figure 4).

Table 3. Goodness-of-fit statistics and parameter estimates of the single- and two-entry additive systems for *Pinus ayacahuite*

Additive system	Variable	R^2_{adj}	*RMSE*	*MAE*	*AIC*	*TRE*	*MPSE*	∑*R*		Estimator	*SE*	*Pr* > |*t*|
Single-entry*	V_{st}	0.975	0.174	0.135	−189.073	−0.076	10.449	12	β_0	0.000608	0.000105	< 0.0001
	V_{br}	0.797	0.047	0.034	−333.845	−0.583	77.334	11	β_1	2.121305	0.045500	< 0.0001
	V_{wt}	0.975	0.189	0.147	−177.261	−0.104	10.941	11	α_0	14.00654	0.906000	< 0.0001
	–	–	–	–	–	–	–	–	α_1	3.093948	0.225700	< 0.0001
Two-entry*	V_{st}	0.993	0.090	0.064	−258.947	−0.672	5.233	6	β_0	0.000068	0.000013	< 0.0001
	V_{br}	0.798	0.047	0.034	−333.967	−1.721	74.058	7	β_1	1.786801	0.034400	< 0.0001
	V_{wt}	0.990	0.119	0.084	−226.202	−0.730	6.767	7	β_2	1.032992	0.079400	< 0.0001
	–	–	–	–	–	–	–	–	α_0	13.679420	0.832400	< 0.0001
	–	–	–	–	–	–	–	–	α_1	3.012850	0.207600	< 0.0001

*The H_0 of the White test is not rejected; V_{st} – stem volume over bark (m^3); V_{br} – branch volume over bark (m^3); V_{wt} – whole-tree volume over bark (m^3); R^2_{adj} – adjusted coefficient of determination; *RMSE* – root mean square error; *MAE* – mean absolute error; *AIC* – Akaike information criterion; *TRE* – total relative error; *MPSE* – mean percent standard error; ∑*R* – relative rank value of each model; α_n, β_n – model parameter estimates; *SE* – standard error; *Pr* > |*t*| – probability of observing the *t*-value under H_0 (smaller values indicate stronger evidence against it)

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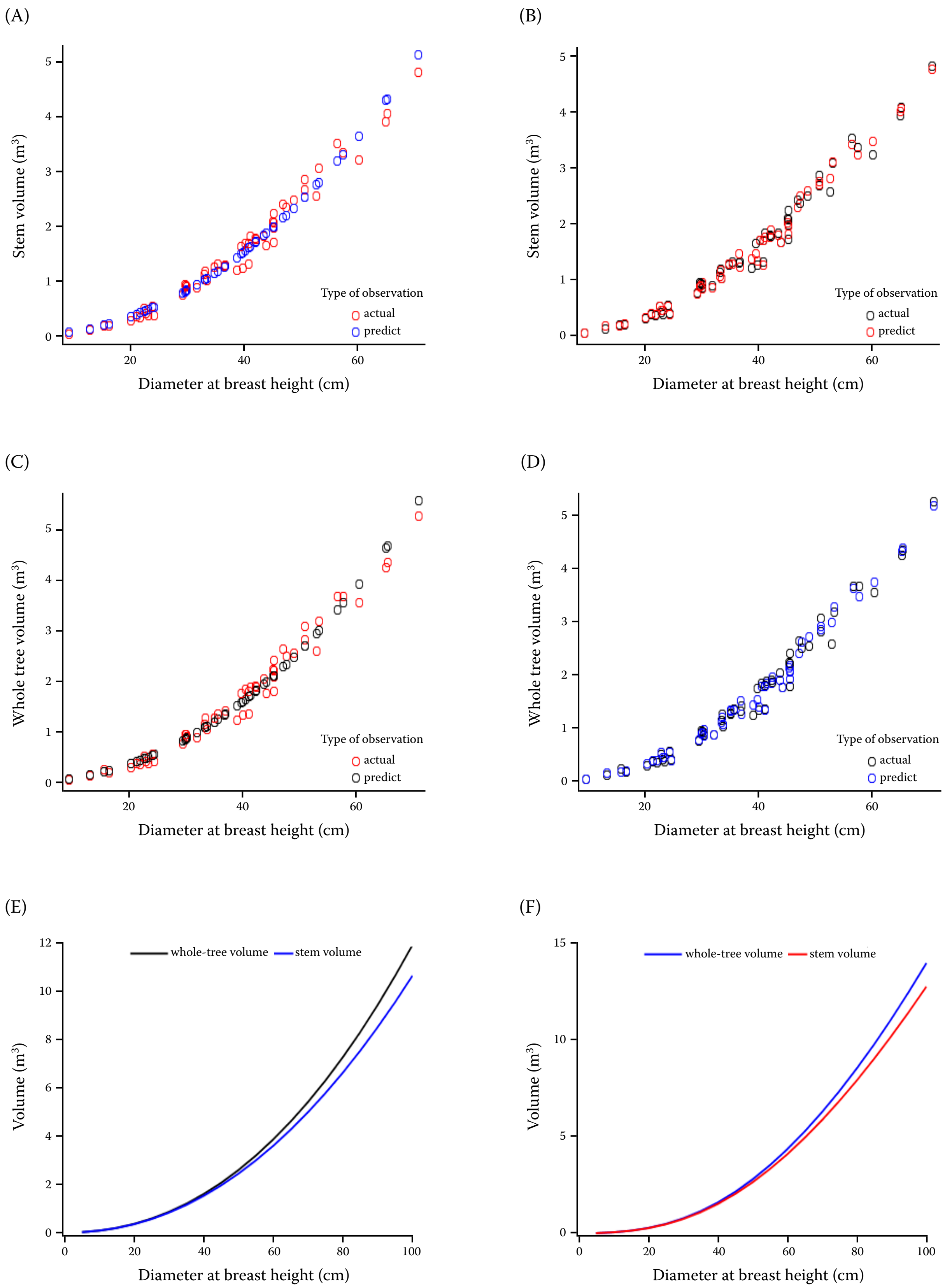

Figure 4. Comparison between predicted and observed volumes for *Pinus ayacahuite*: stem volume (A, B), whole-tree volume (C, D), and the relationship between stem volume and whole-tree volume (E, F), using the single-entry (A, C, E) and two-entry (B, D, F) additive systems, respectively

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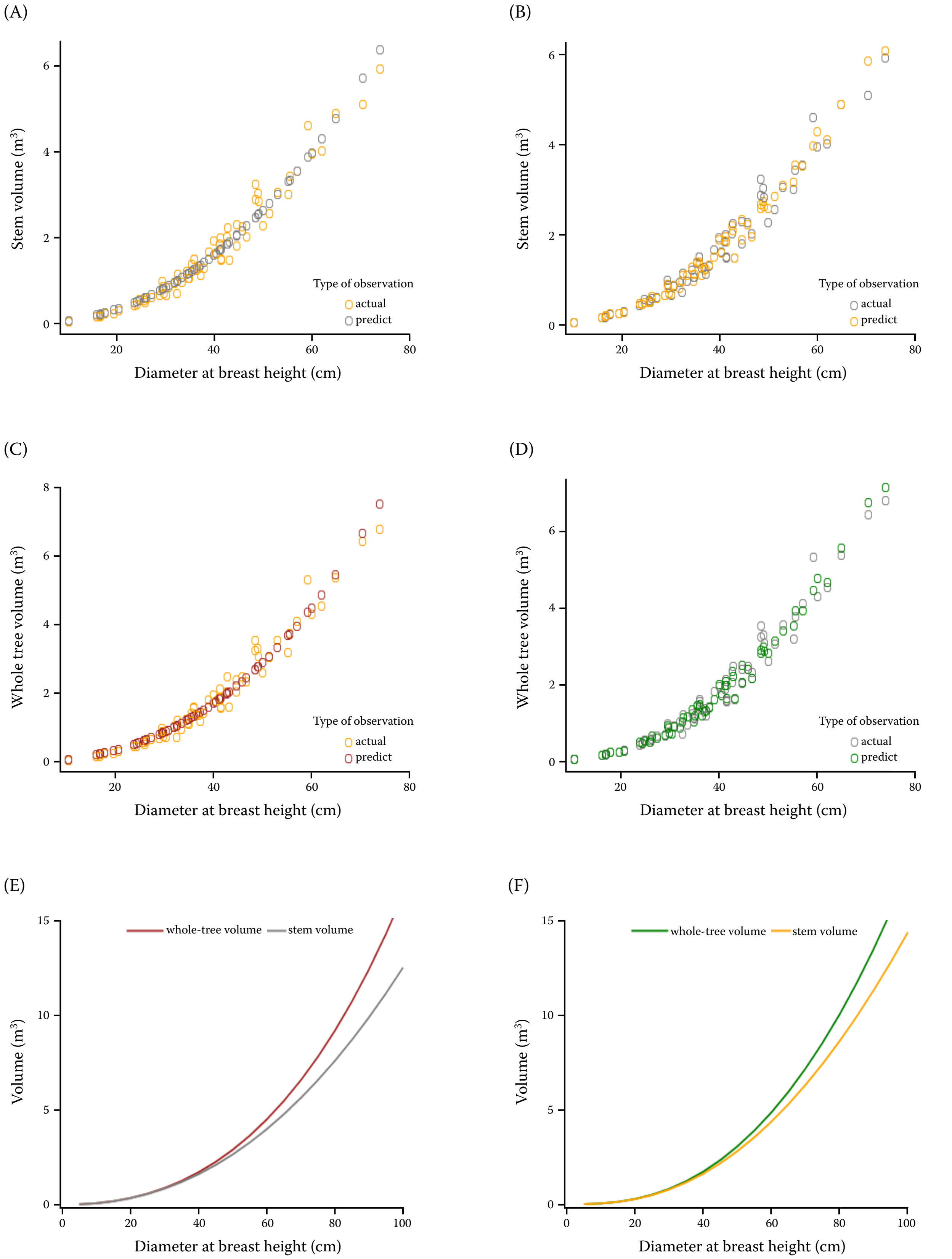

Figure 5. Comparison between predicted and observed volumes for *Pinus douglasiana*: stem volume (A, B), whole-tree volume (C, D), and the relationship between stem volume and whole-tree volume (E, F), using the single-entry (A, C, E) and two-entry (B, D, F) additive systems, respectively

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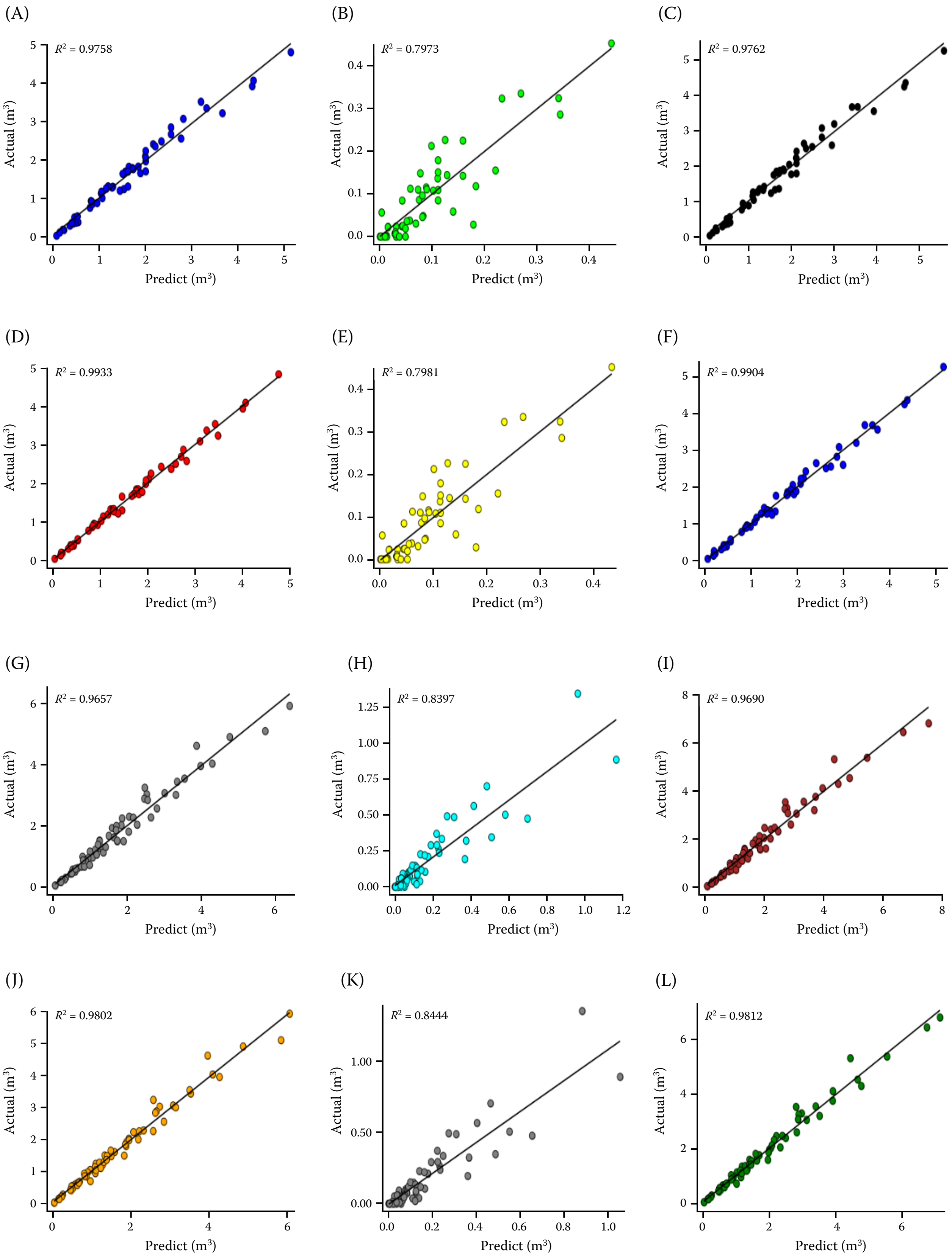

Figure 6. Comparison between observed and predicted values for stem volume (A, D, G, and J), branch volume (B, E, H, and K), and whole-tree volume (C, F, I, and L), using single-entry (A–C and G–I) and two-entry (D–F and J–L) additive volume systems for *Pinus ayacahuite* (A–F) and *P. douglasiana* (G–L), respectively

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Table 4. Goodness-of-fit statistics and parameter estimates of the single- and two-entry additive systems for *Pinus douglasiana*

Additive system	Variable	R^2_{adj}	*RMSE*	*MAE*	*AIC*	*TRE*	*MPSE*	ΣR		Estimator	*SE*	$Pr > \|t\|$
Single-entry*	V_{st}	0.966	0.250	0.182	–177.426	–0.112	12.915	12	β_0	0.000410	0.000087	< 0.0001
	V_{br}	0.839	0.097	0.059	–300.921	3.587	54.699	9	β_1	2.242144	0.054500	< 0.0001
	V_{wt}	0.969	0.277	0.189	–160.765	0.203	12.530	11	α_0	16.852790	0.762600	< 0.0001
	–	–	–	–	–	–	–	–	α_1	3.951537	0.186200	< 0.0001
Two-entry*	V_{st}	0.979	0.195	0.115	–207.641	–0.403	6.546	6	β_0	0.000062	0.000009	< 0.0001
	V_{br}	0.839	0.097	0.057	–300.839	4.617	52.766	9	β_1	1.986211	0.055600	< 0.0001
	V_{wt}	0.981	0.218	0.140	–190.326	0.020	7.304	7	β_2	0.864927	0.090100	< 0.0001
	–	–	–	–	–	–	–	–	α_0	15.814550	1.019800	< 0.0001
	–	–	–	–	–	–	–	–	α_1	3.687032	0.253200	< 0.0001

*The H_0 of the White test is not rejected; V_{st} – stem volume over bark (m^3); V_{br} – branch volume over bark (m^3); V_{wt} – whole-tree volume over bark (m^3); R^2_{adj} – adjusted coefficient of determination; *RMSE* – root mean square error; *MAE* – mean absolute error; *AIC* – Akaike information criterion; *TRE* – total relative error; *MPSE* – mean percent standard error; ΣR – relative rank value of each model; α_n, β_n – model parameter estimates; *SE* – standard error; $Pr > |t|$ – probability of observing the t-value under H_0 (smaller values indicate stronger evidence against it)

Additive systems for *P. douglasiana*. The additive volume systems evaluated for *P. douglasiana* explained between 83.9% and 98.1% of the total observed variability, with all parameter estimates statistically significant ($P < 0.0001$) (Table 4). As observed for *P. ayacahuite*, the branch volume model yielded the lowest adjusted coefficient of determination (R^2_{adj}), with a value close to 84% in both systems.

Based on the total relative rank criterion, the two-entry additive system exhibited the best overall performance (Table 4) and was therefore considered the most suitable approach for volume estimation in this species. The comparison between observed and predicted values revealed a high degree of agreement and followed the expected trend. The relationship between stem volume and whole-tree volume remained biologically coherent, with stem volume consistently lower than whole-tree volume, thereby validating the mathematical and biological consistency of the fitted models. These findings support the reliability of the proposed additive systems for estimating tree volume using either diameter at breast height (*DBH*) alone or in combination with total height (*H*) (Figure 5).

Figure 6 shows the regression fit between observed and predicted volume values for both *P. ayacahuite* and *P. douglasiana*, across stem, branch, and whole-tree components. In both the stem volume and whole-tree volume models, the coefficient of determination (R^2) exceeded 97%, indicating strong agreement between predictions and observed values and demonstrating the robustness and reliability of the fitted systems.

DISCUSSION

In this study, additive volume systems were developed for *P. ayacahuite* and *P. douglasiana* by formulating simultaneous univariate and bivariate equations. The first system employed only diameter at breast height (*DBH*) as a predictor variable, whereas the second incorporated both *DBH* and total height (*H*), recognising the combined influence of these dimensions on tree-volume estimation.

Parameter estimation was carried out using the NLSUR method, which enabled efficient simultaneous fitting of all equations within the system. Additionally, to correct for heteroscedasticity in the residuals, an adaptive weighting scheme was implemented, whereby each observation was assigned a weight inversely proportional to the estimated variance of its error.

Based on the *RMSE* values, the two-entry systems outperformed their single-entry counterparts, reducing the estimation error for whole-tree volume by 37% in *P. ayacahuite* and 21% in *P. douglasiana* (Tables 3 and 4). This behaviour is consistent with findings reported by Ramírez-Martínez et al. (2018) and Monárrez-González et al. (2024),

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who observed significant accuracy improvements when incorporating both *DBH* and *H* into volume equations for coniferous species.

The comparison between observed and predicted values revealed a consistent linear relationship, with coefficients of determination (R^2) exceeding 0.97 for stem and whole-tree volume components, indicating that the models explained a high proportion of the observed variability (Piñeiro et al. 2008). This high level of agreement reflects the robustness and reliability of the obtained estimates. In contrast, the branch component yielded an R^2 of approximately 0.84, which, although acceptable, highlights the inherent difficulty in accurately modelling branch volume in conifers. For example, Guzmán-Santiago et al. (2020) obtained R^2 values ranging from 0.25 to 0.49, while Corral-Rivas et al. (2017) reported R^2 values from 0.64 to 0.70.

The proportion of volume contributed by the branches is relevant for improving the accuracy of forest inventories, especially when non-commercial fractions are included. In *P. ayacahuite*, branches accounted for an average of 5.5% in the best-performing system (two-entry). For *P. douglasiana*, this proportion was higher, at 9.0%. These results are consistent with those of Santiago-García (2025), who estimated that, on average, coarse branches accounted for 6.4% in *P. patula* and 5.6% in *P. pseudostrobus*, also noting an increasing trend in higher diameter classes. A detailed understanding of these fractions improves the accuracy of total volume calculations and enables more precise estimates of the available merchantable wood.

In the two-entry additive system, total height (*H*) proved as an essential variable for improving the precision of volume estimation. However, field measurement of *H* is costly and time-consuming compared to *DBH*. To optimise this process, local, global, and generalised height–diameter models calibrated at the species and study area level may be used (López-Villegas et al. 2017; Santiago-García et al. 2020b). These equations allow for the indirect estimation of *H*, which can significantly streamline the calculation of whole-tree volume and its components.

The volume model used to construct the two-entry additive system corresponds to the Schumacher-Hall equation, which explained more than 98% of the observed variability in the data (Tables 3 and 4). This equation has demonstrated high reliability and precision in estimating the volume of various coniferous and broadleaf species across different regions of the world (Vargas-Larreta et al. 2017; de Lima et al. 2020; de Souza et al. 2024). In recent years, its application has been extended through simultaneous fitting with taper functions for estimating merchantable volume, which has enhanced the characterisation of timber resources (Corral-Rivas et al. 2017; Zhao et al. 2017).

The relative ranking criterion proved to be an objective and effective tool for selecting the best-performing model among the evaluated systems (Poudel, Cao 2013; Ogana, Ercanli 2022). By integrating multiple goodness-of-fit statistics into a unified framework, it identifies the most suitable option based on overall performance rather than individual metrics. In this study, the two-entry additive system consistently achieved the lowest relative rank values, thus confirming its superior predictive ability.

Although two-entry additive systems offer greater accuracy by including *H* as an explanatory variable, the results from the single-entry system should not be disregarded. The single-entry system's main advantage lies in requiring only *DBH* to estimate volume, which simplifies its use in the field. However, its applicability depends on the required level of precision in volume and component estimation (Ramírez-Martínez et al. 2018).

Volume is a key indicator for quantifying forest biomass and assessing the role of forests in the carbon cycle and the provision of ecosystem services. Recent studies have found that volume equations properly calibrated for each species allow for highly accurate estimation of both biomass and stored carbon in trees (Balboa-Murias et al. 2006; Ordóñez-Prado et al. 2024). These equations play a fundamental role in forestry and forest management, as they form the basis for harvest planning, inventory design, and estimation of timber yield (Santiago-García et al. 2020a).

Beyond their statistical robustness, additive volume systems offer several practical advantages: they ensure internal consistency among volume components, reduce the propagation of estimation errors, and allow flexible application across a wide range of tree sizes and stand conditions. The proposed additive systems represent a sound alternative for accurate and biologically consistent volume estimation in *P. ayacahuite* and *P. douglasiana*, and their adoption can significantly strengthen decision-making processes in forestry practice.

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To further improve estimation accuracy, the adoption of artificial intelligence (AI) techniques has emerged as a promising alternative for analysing complex data and identifying non-linear patterns (Huy et al. 2022; Seely et al. 2025). Machine learning algorithms, such as neural networks and random forests, offer the potential to integrate multiple sources of information, including remote sensing data and dendrometric variables, to achieve more accurate predictions (Liu et al. 2023; Santiago-García 2025). Nevertheless, the use of AI in forest management remains in a developing phase, constrained by the need for validation data and the black-box nature of models, which limits their interpretability and acceptance by end users (Damaševičius et al. 2024). In this context, traditional allometric models remain the most practical and extensively validated option for volume estimation, given their simplicity, moderate data requirements, and broad empirical support (Burkhart, Tomé 2012; Zeng et al. 2017; Behling et al. 2019).

CONCLUSION

Single- and double-entry volume systems are essential tools for estimating the volume of standing trees. In this study, the additive double-entry systems based on the Schumacher-Hall model demonstrated superior performance in estimating both whole-tree volume and its structural components; therefore, their preferential application is recommended. The simultaneous fitting of equations significantly improved the goodness-of-fit statistics and the precision of the estimated parameters. The recommended ranges of application for both species are *DBH* between 9 and 75 cm, and *H* between 9 and 34 m. Consequently, the models developed using *DBH* and *H* represent a robust alternative for accurately quantifying timber stocks in the forest harvesting areas of Ixtlán de Juárez, Oaxaca, Mexico.

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